# Essays in Financial Econometrics

by

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Dissertation submitted in partial fulfillment of the requirements for the degree of
Doctor of Philosophy in the Department of Economics
in the Graduate School of Duke University
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# Abstract

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# Abstract

This dissertation consists of three essays. In the first essay, I analyze the performance of five different classes of integrated variance estimators when applied to various stocks of differing market capitalization in an attempt to discover the circumstances under which one estimator should be chosen over another. In recent years, there has been an explosion of research on the volatility of stock returns. As high frequency stock price data became more readily available, there have been many proposed estimators of integrated variance which attempt to take advantage of the informational gains of high-frequency data while minimizing any potential biases that arise from sampling at such a fine scale. These estimators rely on various assumptions about the price process which can make them difficult to compare theoretically. I find that across several stocks in different size deciles, the truncation estimator outperforms the other estimators of integrated variance. Furthermore, I find that choosing a truncation parameter of 2-3 standard deviations leads to the most accurate estimates on average.

In the second essay, I estimate latent factor models of liquidity and volatility. Common liquidity and volatility factors are extracted using multiple liquidity and volatility measures. Additionally, latent factors are extracted by aggregating across both liquidity and volatility resulting in what we will call the common "uncertainty" factors. This underlying uncertainty factor is correlated with the individual and common liquidity and volatility factors as well as returns. I find that the under-

lying uncertainty risk factor is significantly priced in the cross section of expected returns, while the risks associated solely with liquidity and volatility are not. These results suggest that the liquidity risk and volatility risk may both proxy for an underlying uncertainty risk which drives the significant results when considering them individually.

The third essay further explores the "uncertainty" factor and links it to the macroeconomy with the hope of accurately forecasting real GDP growth, growth in industrial production, and growth in the unemployment rate. I show that shocks to the uncertainty factor have both in- and out-of-sample predictability for real GDP growth as well as growth for both industrial production and unemployment rate. While the uncertainty factor significantly improves forecast performance over an AR(1) model, there is no indication that the forecasts based on our uncertainty factor significantly outperform forecasts based on an aggregate liquidity measure.

To my loving wife and daughter.

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# 1

# Data-Based Ranking IV Estimators Across Size Deciles

## 1.1 Introduction

While early work in realized measures such as Merton (1980) and Zhou (1996) recognized the benefits of utilizing higher frequency data to measure variability over a longer period, only recently has high-frequency intraday price data become available. Subsequently, over the past several years with the increased availability of high-frequency stock data, there has been a strong research focus on the best ways to exploit this increase in information. One specific use of high-frequency data on which there has been a strong focus is that of measuring price volatility, or quadratic variation. Realized variance (RV), or the sum of squared intraday returns (see Andersen et al. (2001) and Barndorff-Nielsen and Shephard (2002)), has been the launching pad for many other estimators of quadratic variation that utilize high-frequency returns data. Many of these have sought to reduce microstructure noise, isolate the continuous component of volatility, reduce finite sample bias, or otherwise improve upon our ability to measure the variation of asset prices[1].These different estimators are often based on different assumptions about the price process. Additionally, one may be based upon sampling in calendar time while another utilizes tick time sampling. These, as well as other tractability issues, often prohibit the theoretical asymptotic comparison of these various estimators.

One shortcoming of the realized variance estimator is that it is only a measure of the *total* variation of the price process, or quadratic variation. For years, research in finance has been based on a continuous price process, however, recently it has become clear that price processes are better represented as continuous brownian motion with jumps[2]. In many fields, e.g. risk management, options pricing and volatility forecasting, it can be useful to obtain an estimate of the continuous component of quadratic variation, or integrated variance (IV), without including the variation caused by jumps. The need to isolate and estimate the integrated variance in the presence of possible jumps has lead to the development of various estimators which attempt to exclude jump variation[3] as well as tests to determine if and when jumps may have occurred in the data[4]. With a variety of possible IV estimators available it is useful to obtain a better understanding of which measure to use when investigating a specific empirical question. Since it is often difficult to theoretically compare the different estimators, we must rely on empirical methods to determine the most appropriate estimator for any given asset.

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[1] See Barndorff-Nielsen and Shephard (2004), Mancini (2009), Lee and Mykland (2008), Andersen et al. (2008), and **?**. Additional estimators that use subsampling techniques include Zhang (2006), Zhang et al. (2005). For pre-averaging methods see Jacod et al. (2009)and Podolskij and Vetter (2009). Barndorff-Nielsen et al. (2009) develop kernel-based autocovariance adjustments to reduce the effects of noise.

[2] See, for example, Andersen et al. (2002), Bates (2000), Chan and Maheu (2002), Chernov et al. (2003), Eraker (2004), and Eraker et al. (2003).

[3] See Barndorff-Nielsen and Shephard (2004), Mancini (2009), Andersen et al. (2008), Christensen et al. (2010), Aït-Sahalia and Jacod (2012), and **?**

[4] See Barndorff-Nielsen and Shephard (2004), Huang and Tauchen (2005), Barndorff-Nielsen and Shephard (2006), Andersen et al. (2007) and Aït-Sahalia and Jacod (2012) among others.

Recently, Patton (2011a) developed an empirical method that allows for the ranking of various estimators when the true underlying process is unobserved, as is the case in the volatility of asset prices. In his implementation of the empirical ranking using high-frequency returns for IBM, Patton (2011a) finds that for the simple RV estimator it is optimal to use a sampling frequency between 15 seconds and 5 minutes. This technique has also been used by Patton and Sheppard (2009) who rank various estimators of quadratic variation and find it is often optimal to use a combination of estimators as opposed to simply choosing a single estimator. This paper utilizes similar techniques to examine the performance of IV estimators for stocks across different size deciles.

Studies that estimate the integrated variance for specific assets often focus their attention on the stocks with the largest market capitalization[5]. We chose to look over a wide range of stocks from different size deciles in order to explore how the various estimators perform on stocks with varying liquidity. Specifically, this study will be conducted over 30 stocks from the NYSE. Once all of the stocks were sorted into their size decile, the top ten stocks from the tenth, sixth, and second deciles (where the tenth decile contained the largest stocks on the NYSE) were selected for the study. Such a wide array of stocks will allow for the examination of the relative performance of the estimators on assets of varying size and liquidity. This paper looks to expand upon the findings of Patton (2011a) and Patton and Sheppard (2009) by comparing various IV estimators over a wide range of sampling frequencies and stocks.

The rest of the paper is organized as follows. Section 2 describes the empirical ranking method of Patton (2011a) in further detail. Section 3 discusses the various IV estimators in more detail. A detailed description of the data, including stock selection, cleaning methods, and summary statistics is included in Section 4. Section

[5] For example, **?** and Christensen et al. (2010). Additionally, in the ranking procedures of Patton (2011a) and Patton and Sheppard (2009) only data for IBM was examined.

5 presents the results of both pairwise comparisons as well as tests for the best estimator among a large set of possibilities. Section 6 concludes.

## 1.2 Ranking Method

One main difficulty in comparing measures of integrated variance is that the true IV is unobservable. This difficulty, coupled with the abundance of possible sampling frequencies and estimators, has lead to the need to better understand which choice of sampling frequency and tuning parameters should be used in any given empirical analysis. The empirical techniques presented in Patton (2011a) allow for the comparison of different realized measures even when the true underlying process is unobservable. By proving various moment and distributional conditions, he is able to appeal to existing volatility forecast literature in order to compare the various RV estimators.

In addition to being able to circumvent problems arising from measuring the accuracy of estimators to an unobservable target, these data-based techniques also allow for correlation between microstructure noise and the price process and are straightforward to implement (in part because there is no need to calculate integrated quarticity or the variance of the noise process). However, it is necessary to have a proxy of the true process that is conditionally unbiased in finite samples. Simulation results will be presented in Section 4 to justify the choice of proxy.

Let $\theta_t$ denote the latent IV process which we are interested in measuring as accurately as possible by choosing any one of a possible $k$ realized measures, $X_{i,t}$ for $i = 1, 2, ..., k$. In order to determine the whether one estimator is expected to be more accurate than another, we are interested in measuring

$$E[\Delta L(\theta_t, \mathbf{X}_t) \equiv E[L(\theta_t, X_{i,t})] - E[L(\theta_t, X_{j,t})] \quad \text{for } i \neq j \tag{1.1}$$

where $L(\cdot, \cdot)$ is any predetermined distance measure. The work of Patton (2011a) is to provide a way in which this value can be accurately estimated even though the process $\theta_t$ is unobservable.

In order to begin the ranking process, one needs to choose the distance measure to be used when determining the "most accurate" IV estimator. Any robust pseudo-distance measure (see Patton (2011b)) of the form

$$L(\theta, X) = \tilde{C}(X) - \tilde{C}(\theta) + C(X)(\theta - X) \tag{1.2}$$

where $\tilde{C}(X)$ is the anti-derivative of the decreasing, twice differentiable function C(X) can be chosen when using this data ranking technique. While Patton (2011a) and Patton and Sheppard (2009), consider both the QLIKE and MSE loss functions, MSE was chosen as the loss function in this paper's ranking of IV estimators. The motivation for this is that the QLIKE function doesn't readily accommodate estimates of zero[6]. The MSE loss function can easily manage zero values and there is no need to manipulate the data and replace the zero estimates with an average/minimum value, potentially complicating inference. Additionally, MSE is a generally accepted and widely used distance measure. The specific functional forms of MSE is

$$\text{MSE} \qquad L(\theta, X) = (\theta - X)^2 \tag{1.3}$$

Since $\theta_t$ is unobservable, we must use a proxy, $\tilde{\theta}$, to consistently estimate the difference in average accuracy of different estimators. In order to obtain accurate results, this proxy must be conditionally unbiased for a finite number of observations $N$. Once a finitely unbiased proxy has been selected, Patton (2011a) shows that using an average of leads of the proxy, $Y_t = \sum_{j=1}^{J} \tilde{\theta}_t$, it is possible to obtain estimates of $E[\Delta L(\theta_t, \mathbf{X}_t)$. These results do require one of two assumptions about the true process $\theta_t$; it must either be a random walk or an AR(p) process. In this study, we hold to the assumption that the true integrated variance process follows a random

[6] It is frequently the case, especially with the small stocks in the early half of the sample, that estimates of integrated variance will be 0.

walk. The random walk assumption is supported empirically in work by Andersen et al. (2007) and Hansen and Lunde (2013). Further support comes from the results of Patton (2011a) and Patton and Sheppard (2009) where there was little difference between the random walk case and the AR(p) case. If the random walk assumption holds, then

$$\mathrm{E}[\Delta L(\theta_t, X_t)] = \mathrm{E}[\Delta L(Y_t, X_t)] \tag{1.4}$$

With a couple of additional technical assumptions, then it can be shown that

$$\sqrt{T}\left(\frac{1}{T}\sum_{t=1}^{T}\Delta L(Y_t, X_t) - E[\Delta L(\theta_t, X_t)]\right) \xrightarrow{d} N(0, \Omega_1) \tag{1.5}$$

as $T \to \infty$. Additionally, the conditions that allow for the use of the stationary bootstrap will be met. These results allow for testing multiple estimators using the methods of Hansen et al. (2011) and Giacomini and White (2006) which will be the primary testing methods we consider[7].

## 1.3 IV estimators

While realized variance based on 5 minute returns is generally accepted as an accurate measure of quadratic variation, it is often useful to isolate the integrated variance, or variance due only to the continuous component of the price process. This has led to the development of various realized measures that attempt to isolate the continuous variation while filtering out the variation due to jumps in the price process. This leaves researchers and practitioners with both a variety of choices of estimators and a wide range of possible sampling frequencies when attempting to measure the integrated variance of an asset. Before we continue with a presentation of the results, further discussion about each estimator under consideration is necessary. For notational purposes, let us consider a jump-diffusive price process where

[7] White (2000) and Romano and Wolf (2005) also developed techniques to test multiple models simultaneously.

the logarithmic price, $P_t$, is observed at $N+1$ discrete points throughout the trading day.

The first estimator of daily integrated variance is the Bipower Variation (BV) estimator of Barndorff-Nielsen and Shephard (2004). One benefit to this estimator is that it is a consistent estimator of IV under the assumption of no market microstructure noise but otherwise general conditions. The specific form that it takes is similar to that of the RV estimator, but instead of using squared returns it uses the product of neighboring absolute returns. It has the following functional form

$$BV_N = \frac{\pi}{2}\frac{N}{N-1}\sum_{i=1}^{N-1}|\Delta P_t||\Delta P_{t+1}| \tag{1.6}$$

where $\Delta P_t = P_t - P_{t-1}$. The intuition behind this estimator is as follows. Because there are finitely many jumps during a trading day, as the sampling frequency goes to zero (or as $N \to \infty$), there will not be two jumps in any two subsequent returns. Additionally, the diffusive return will go to zero. For illustrative purposes, consider two returns $\Delta P_t$ and $\Delta P_{t+1}$ where the first return contains a jump and the second does not. The returns when there is no jump, or $\Delta P_{t+1}$ in this case, will go to zero as the sampling frequency gets small. This will cause the product $|\Delta P_t||\Delta P_{t+1}|$ to go to zero and thus eliminate any variation in returns due to the jump component of the price process. One shortcoming of this estimator, however, is that it is biased in finite samples. This bias arises from the fact that in finite samples the diffusive return $|\Delta P_{t+1}|$ does not equal zero and thus the jump return $|\Delta P_t|$ is not completely cancelled out. This drives up the estimated value of IV and creates an upward bias.

A recent attempt to extend upon the thought underlying the BV estimator while minimizing any potential finite sample bias are the MinRV and MedRV estimators of **?**. These estimators are shown to be consistent IV estimators and are more robust to finite jumps in finite samples. The functional forms of the two estimators are as follows:

$$\mathrm{MinRV}_N = \frac{\pi}{\pi - 2}\frac{N}{N-1}\sum_{i=1}^{N-1}\min(|\Delta P_t|, |\Delta P_{t+1}|)^2 \tag{1.7}$$

$$\mathrm{MedRV}_N = \frac{\pi}{6 - 4\sqrt{3} + \pi}\frac{N}{N-2}\sum_{i=1}^{N-1}\mathrm{med}(|\Delta P_{t-1}|, |\Delta P_t|, |\Delta P_{t+1}|)^2 \tag{1.8}$$

The intuition behind these estimators is similar to that of bipower variation; these estimators seek to eliminate the variation in returns due to jumps by taking either the minimum or the median return over a small block size of two or three returns. The jump robustness of the MedRV and MinRV estimators of **?** in relation to bipower variation is that the variation due to the jump return will be completely eliminated by the minimum or median operator. These estimators do rely on the assumption of a constant variance over each block of returns, and by using block sizes of only two or three returns, the MinRV and MedRV estimators are less vulnerable to bias due to intraday volatility patterns.

The fourth IV estimator we will consider in this analysis is the truncation-type estimator, see Mancini (2009), Jacod (2008), and Aït-Sahalia and Jacod (2012).

$$TRV = \sum_{i=1}^{N}(\Delta P_t)^2 1_{\{|\Delta P_t| < cN^{-\bar{\omega}}\}} \tag{1.9}$$

where $\bar{\omega} \in (0, 0.5)$. In the specific implementation of the estimator, we follow Christensen et al. (2010) and set $\bar{\omega} = 0.47$. This form of estimator relies on filtering out returns that exceed a threshold chosen by the researcher. Through these means, the large returns that are the result of jumps in the price process will be eliminated and ideally the only returns that will be considered are diffusive returns. One potential difficulty with truncation estimators is in the choice of threshold. By selecting a different truncation thresholds for the comparison, we hope to gain further insight into the extent to which the choice of the truncation parameter can effect the results.

## 1.4 Implementation

### *1.4.1 Data*

For the purposes of a more thorough ranking, as well as to discover if any estimators may generally perform better for one class of stock over another, the empirical rankings are done over a collection of 30 different stocks. The stocks were chosen based on their market capitalization for the year 2007. All NYSE stocks were sorted into ten size deciles. The ten largest stocks from the tenth (large cap), sixth (mid cap), and second (small cap) size deciles (with the tenth being the largest stocks and the first being the smallest) were then chosen as our sample. Stocks from a range of size deciles were chosen to examine how liquidity and market capitalization may affect the relative performance of the estimators. It may well be the case that a specific estimator outperforms in highly liquid stocks while another shines when used on less liquid ones. Two additional requirements were that the available data for the stock dates to at least 2002, and that the ticker symbol corresponded to an actual company (e.g. mutual funds were ignored). The thirty stocks that were chosen, the total sample size for each stock, as well as some descriptive statistics are presented in Table 1.1.

The TAQ trades data was then cleaned in a method very similar to that outlined in Barndorff-Nielsen et al. (2009). Observations were filtered based on time stamp so that only those occurring between 9:30am and 4:00pm were included. Any zero price was removed and only trades that occurred on the NYSE were included in the final dataset. All entries that had been corrected (or had the variable CORR $\neq 0$) as well as any observation with an abnormal *sale condition* were removed. If there were multiple prices for a single timestamp, then the median price was used for that second. The last recorded price was used for any second on which there was no recorded trade. From this dataset, any desired sampling frequency is readily obtained. Additionally, returns do to stock splits and overnight returns are set to zero.

### *1.4.2 Sampling Frequencies and Tuning Parameters*

For a few of the estimators, the only parameter that one must choose is the sampling frequency. When choosing the sampling frequency, there is always a trade-off between added information and possible noise contamination. The sampling frequencies under consideration are 10 and 30 seconds and 1, 2, 5, 10, 15, 20, and 30 minutes. Again, when varying the sampling frequency, we set the truncation parameter to $6\sqrt{IV}$. When changing the truncation parameter, we fix the sampling to be at 2 minutes.

### *1.4.3 Choice of Proxy*

As was previously mentioned, when implementing the data-based ranking technique of Patton (2011a), it is necessary to have a proxy of the latent variable that is unbiased in finite samples. If an unbiased proxy were unavailable, then the equality of Equation 1.4 would not hold. In the case of quadratic variance, 5-minute RV is widely accepted as an unbiased proxy in finite samples of quadratic variance. In the case of integrated variance, the choice of proxy is not so obvious. The final decision on the choice of proxy was based the simulation results of both **?** and Christensen et al. (2010).

In the simulation study of **?**, they consider six different models for the price process and they compare the finite sample properties of the RV, BV, MinRV, MedRV, Tripower variation (TV), and QRV estimators. They also compare the estimators over a range of sampling frequencies: 12 seconds, 1 minute, and 5 minutes. In each of the six models, the price process, $\{P_t\}$, follows a brownian motion without drift and with instantaneous volatility $\sigma(t)$:

$$dP(t) = \sigma(t)dW_1(t) \tag{1.10}$$

They calibrate the IV to an annualized volatility of 20% and sample at a two second frequency. For the second model, they add a U-shaped pattern into the intraday volatility path to match that which is commonly seen in the data. In Model 3, the authors sample randomly (without replacement) from a grid of one second returns. Jumps are added into Models 4 and 5 by introducing an additional term to the log-price process. For these models, it will follow the path

$$dP(t) = \sigma(t)dW_1(t) + dJ_t \tag{1.11}$$

where $J_t$ is a Poisson jump process that is independent of the brownian motion. A simple brownian motion with Gaussian i.i.d. noise is generated in Model 6. Since we are interested only in the unbiased nature of the estimators, only the relative bias, $(\frac{\hat{IV}}{IV})$, results are reproduced in Table 1.2 where $\hat{IV}$ denotes the estimate of the true IV.

For each model, a total of 2,500 days are simulated and the relative bias is calculated as the average $\frac{\hat{IV}}{IV}$ across the entire 2,500 day simulation. Examining Table 1.2, we see that at the 1 minute sampling frequency, the relative bias for MedRV is between 1.026 and 0.990. This tells us that across all of the six models under consideration, the MedRV estimate of the true integrated variance is no more than 2.6% larger than and 1% smaller than the true value on average. Furthermore, Christensen et al. (2010) find that for 100,000 simulations from 10 different models, the expected relative bias for MedRV is 1 for 6 of the models under consideration. It is no more than 1.03 for 3 of the remaining 4 models. The only model for which MedRV is largely biased is when the price process follows a brownian motion with an outlier (represented by 2 consecutive jumps of opposite signs – a violation of an underlying assumption of MedRV). From these simulation results, the proxy used for the ranking technique was chosen to be the MedRV estimator. It was sampled at the 2 minute frequency based on the simulations and following the advice of **?**.

## 1.5 Results

### *1.5.1 Truncated Realized Variance*

Before examining the formal statistical tests, it is worthwhile to visually examine the performance of the truncation estimator as the truncation parameter changes. By plotting the average loss as a function of the truncation parameter we are able to identify the threshold that will provided the lowest average loss for a specific stock. Similar to volatility signature plots, see Andersen et al. (2000), Figures 1.1 - 1.3 provide a visual means of selecting the optimal threshold parameter. In each of these plots, the sampling frequency is fixed at 2 minutes and the truncation parameter is allowed to vary. The graphs in Figures 1.1 - 1.3 plot the average MSE loss as a function of the truncation parameter (number of standard deviations above which a return is considered a jump). The plot with the crosses is using the 2 minute MedRV as the proxy while the dotted line is the average MSE loss using 2 minute BV as the loss function. The value at which the plots obtain a minimum is the value of the threshold that will yield the lowest average MSE loss for any given stock. In almost all of the individual plots for the 30 stocks we see the minimum occurring at either 2 or 3. This result is robust across each of the size deciles under consideration, and indicates that the result is quite robust. While not included in the plots in order to keep them as clean as possible, this result is robust to also using 2 minute MinRV as the proxy.

### *1.5.2 Giacomini and White (2006) tests*

In this section, we implement Giacomini and White (2006) tests on whether two competing IV estimators have equal average accuracy conditional on the same information set, $\mathcal{F}_{t-1}$. The specific null hypothesis in question is

$$H_0: \qquad E[L(\theta_t, X_{t,i})|\mathcal{F}_{t-1}] - E[L(\theta_t, X_{t,j})|\mathcal{F}_{t-1}] = 0 \tag{1.12}$$

The results of Patton (2011a) allow this test to be run using a simple regression of the following form

$$L(\theta_t, X_{t,i}) - L(\theta_t, X_{t,j}) = \beta_0 + e_t \tag{1.13}$$

These regressions were run for each pairing of estimators based on the 2 minute sampling frequency. As discussed previously, the proxy used for the regressions is the MedRV estimator based on data sampled at the five minute frequency. The resulting robust t-statistics are reported for each stock in Table 1.3 – Table 1.32.

Each of the tables is constructed in this manner. The column variables reference the $X_{t,j}$ variable from the regression equation and the $X_{t,i}$ is listed in the row labels. Thus, a positive value indicates that the average accuracy of the column variable is superior to the average accuracy of the row variable. Consider Table 1.5. In this table, the $\text{TRV}_3$ column consists of positive, statistically significant t-statistics. This means that in pairwise comparisons with each of the other estimators (each sampled at the 2 minute frequency), the truncated realized variance estimator with a truncation parameter of 3 standard deviations (see Equation 1.9) has a better average accuracy than each of the other proposed estimators. Similarly, the upper right value of 2.90 implies that 2 minute BV is significantly more accurate on average than 2 minute RV (this is what we would hope to see since we are attempting to ignore jumps in our estimation of integrated volatility).

One interesting trend that emerges for the 10 stocks in the largest decile is that the 2 minute RV estimator is rarely out-performed by the other estimators. However, this trend does not hold for the stocks in the middle and lower deciles. For nearly all the stocks in the middle size decile, 2 minute RV is significantly worse at estimating the integrated variance than each of the estimator specifically developed to estimate IV. The poor performance of RV in pairwise comparisons continues into the lower decile as well. This discrepancy between stocks in the largest size decile and those in the lower deciles is most likely explained by the discrepancy in the average number of trades between large and small stocks. As the summary statistics in Table 1.1 indicate, the largest stocks trade at a much higher frequency than the smaller stocks. As a result, small frequent jumps are possibly more likely to appear in the large stocks[8]. This would lead to a smaller difference between the RV estimator and any of the IV estimators since it would be more difficult for the later to distinguish jump returns from returns on the continuous process. The trading activity drops off significantly for the smaller stocks possibly allowing the IV estimators to more easily distinguish jump returns from non-jump returns.

Another common trend that occurs in just over half of the stocks across all deciles is that the $TRV_3$ estimator significantly outperforms the other estimators for the 2 minute sampling frequency. Furthermore, only once (out of the 130 pairwise comparisons for $TRV_3$) is $TRV_3$ significantly worse than the other estimators. In that single instance, for the small stock RHB (see Table 1.32), it is found to be significantly worse than MinRV. Finally, examining each of the pairwise comparisons for the 2 minute BV estimator reveals that the only estimator to ever significantly outperform BV is the $TRV_3$. This is consistent across each of the size deciles. The data suggests that the easily implemented BV estimator performs quite well across a variety of stocks and the additional refinements may not be as helpful when applied to actual stock data as opposed to simulated data.

### *1.5.3 Set of Best Estimators*

While pairwise comparisons can be informative, it can be difficult to test every possible specification of the estimators in a pairwise manner. The model confidence set (MCS) procedure of Hansen et al. (2011) is implemented in order to test which

[8] For example, the empirical results of Aït-Sahalia and Jacod (2011) suggest the presence of infinite-activity jumps in the two large stocks they test, Microsoft (MSFE) and Intel (INTC).

estimators, sampling frequency, and tuning parameters are significantly better than the others. While the MCS may not yield a single estimator as being the best, it will result in a set of estimators which will contain the best estimator with a specific level of confidence. This procedure will allow the data to determine which estimator(s) most accurately estimate IV.

There are a total of 62 different estimators under consideration. For each estimator (BV, MedRV, MinRV, and $TRV_6$) we have 9 possible sampling frequencies ranging from 10 seconds to 30 minutes. Additionally, we fix the sampling frequency of TRV at 2 minutes and then vary the truncation parameter to take on one of the following values 1, 2, 3, 4, or 5 standard deviations.

The results are summarized in Tables 1.33 - 1.35. The tables are divided into the three size groups that we are considering. The model confidence set is determined for the IV estimators for each stock, and the tables indicate the number of times that a specific estimator was in the model confidence set. For each group of stocks, there were 2 or 3 stocks which included all of the estimators in the model confidence set. Another noticeable trend was that the TRV with a truncation of 2 or 3 standard deviations was almost always selected as one of the best estimators for integrated variance regardless of the size decile. Additionally, bipower variation (BV), specifically the 2 minute frequency, was frequently selected as belonging to the model confidence set with 95% confidence.

In section 5.1, we found that the truncation estimator had the lowest average MSE loss when the truncation parameter was set to either 2 or 3 standard deviations. The presence of this same estimator in the model confidence set of virtually every stock further confirms the performance of TRV using a truncation of 2 or 3 standard deviations. In section 5.2, the pairwise Giacomini and White (2006) tests indicated that only TRV was able to outperform 2 minute BV across all of the stocks. The presence of 2 minute BV in 24 out of 30 model confidence sets further supports the idea that in most cases, the simple bipower variation estimator will be an optimal choice of estimator.

## 1.6 Conclusion

The recent availability of high frequency asset returns has lead to the development of a numerous variety of volatility measures. With the current number of estimators and no feasible way to compare them theoretically, researchers are always confronted with the question of which estimator will provide the most accurate IV measure for a given asset. In addition to choosing the estimator, and any subsequent tuning parameters, one must also select a sampling frequency for the data. This paper applies the data-based ranking technique of Patton (2011a) to estimators of integrated variance. It focuses on stocks over a wide range of sizes and, as a result, also liquidities. The results indicate that if one is interested in measuring the integrated variance, then TRV with a threshold of 2 or 3 standard deviations not only has the lowest average MSE loss for the truncation parameters, but is almost always included in the set of "best" estimators of integrated variance for a stock regardless of its market size and trading volume. In pairwise Giacomini and White (2006) tests, $TRV_3$ significantly outperforms the other estimators (sampled at a two minute frequency) for the majority of the 30 stocks included in the analysis. Further examination of the results from pairwise testing and from the model confidence set procedure, indicates that bipower variation is often a "best" estimator of IV. While there doesn't appear to be a one-size-fits-all solution for estimating integrated variance, both BV (1-2min) and the truncated estimator do well in a variety of situations. Additionally, this paper provides a quick, visual method for determining the optimal truncation threshold for any given asset, similar to volatility signature plots.

## 1.7 Tables and Figures

Table 1.1: Summary Statistics for the 30 selected stocks

| Ticker | Year Range | Average # trades per day | Average # trades per day 1993-2002 | Average # of trades per day 2003-2012 |
|---|---|---|---|---|
| AIG | 1993-2012 | 3,942.3 | 927 | 7,092 |
| BAC | 1993-2012 | 6,524.7 | 1,119 | 12,171 |
| GE | 1993-2012 | 6,341.8 | 2,071 | 10,803 |
| IBM | 1993-2012 | 4,689.8 | 1,737 | 7,774 |
| JNJ | 1993-2012 | 4,441.3 | 1,113 | 7,918 |
| MO | 1993-2012 | 3,544.4 | 1,323 | 5,864 |
| PFE | 1993-2012 | 4,889.3 | 1,672 | 8,250 |
| PG | 1993-2012 | 4,264.5 | 1,058 | 7,614 |
| T | 1993-2012 | 4,463.0 | 1,394 | 7,660 |
| WMT | 1993-2012 | 4,937.6 | 1,225 | 8,816 |
| AKS | 1995-2012 | 1,936.7 | 146 | 3,384 |
| ATW | 1997-2012 | 944.3 | 132 | 1,402 |
| BPL | 1993-2012 | 131.37 | 22 | 246 |
| CTV | 1997-2011 | 1,176.1 | 209 | 1,567 |
| CW | 1993-2012 | 299.7 | 21 | 571 |
| DRQ | 1997-2012 | 622.6 | 59 | 927 |
| HXL | 1993-2012 | 633.5 | 33 | 1,259 |
| KEX | 1996-2012 | 478.9 | 49 | 758 |
| RBA | 1998-2012 | 312.12 | 15 | 459 |
| VMI | 2002-2012 | 487.8 | 95 | 501 |
| CCC | 1993-2012 | 380.9 | 39 | 738 |
| CIA | 2002-2012 | 144 | 44 | 153 |
| CMO | 1993-2012 | 327.8 | 85 | 581 |
| CRN | 1998-2010 | 156.2 | 18 | 186 |
| CV | 1993-2012 | 82.23 | 22 | 144 |
| CYD | 1994-2012 | 208.4 | 8 | 360 |
| DCO | 1996-2012 | 96.4 | 35 | 136 |
| HGR | 1998-2012 | 263.7 | 83 | 347 |
| MIG | 1995-2012 | 189.2 | 12 | 317 |
| RHB | 1998-2012 | 316.4 | 96 | 383 |

Table 1.2: Simulation Results from Andersen et al. (2012)

Relative Bias for 12 sec. sampling frequency

| | RV | BV | TV | QRV | MinRV | MedRV |
|---|---|---|---|---|---|---|
| Model 1:BM | 1.000 | 1.000 | 1.000 | 0.999 | 1.000 | 1.000 |
| Model 2: SV-U | 0.999 | 0.999 | 0.998 | 0.971 | 0.999 | 0.998 |
| Model 3: BM + Sparcity | 0.999 | 0.974 | 0.965 | 0.969 | 0.955 | 0.962 |
| Model 4: BM + 1 Jump | 1.244 | 1.018 | 1.009 | 1.001 | 1.002 | 1.002 |
| Model 5: BM + 4 Jumps | 1.250 | 1.035 | 1.020 | 1.006 | 1.006 | 1.006 |
| Model 6: BM + Noise | 1.078 | 1.079 | 1.079 | 1.078 | 1.079 | 1.079 |

Relative Bias for 1 min. sampling frequency

| | RV | BV | TV | QRV | MinRV | MedRV |
|---|---|---|---|---|---|---|
| Model 1:BM | 1.001 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Model 2: SV-U | 0.995 | 0.993 | 0.990 | 0.969 | 0.993 | 0.991 |
| Model 3: BM + Sparcity | 1.001 | 0.993 | 0.991 | 0.988 | 0.988 | 0.990 |
| Model 4: BM + 1 Jump | 1.242 | 1.038 | 1.023 | 1.006 | 1.007 | 1.007 |
| Model 5: BM + 4 Jumps | 1.250 | 1.073 | 1.051 | 1.026 | 1.023 | 1.026 |
| Model 6: BM + Noise | 1.003 | 1.004 | 1.003 | 1.003 | 1.004 | 1.004 |

Relative Bias for 5 min. sampling frequency

| | RV | BV | TV | QRV | MinRV | MedRV |
|---|---|---|---|---|---|---|
| Model 1:BM | 1.001 | 1.001 | 1.002 | 1.001 | 1.002 | 1.002 |
| Model 2: SV-U | 0.990 | 0.979 | 0.968 | 0.967 | 0.979 | 0.969 |
| Model 3: BM + Sparcity | 1.002 | 1.001 | 1.003 | 0.998 | 1.000 | 1.002 |
| Model 4: BM + 1 Jump | 1.241 | 1.075 | 1.053 | 1.031 | 1.024 | 1.027 |
| Model 5: BM + 4 Jumps | 1.251 | 1.131 | 1.107 | 1.086 | 1.073 | 1.082 |
| Model 6: BM + Noise | 1.002 | 1.004 | 1.004 | 1.001 | 1.005 | 1.002 |

Table 1.3: GW test t-statistics for AIG (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 0.97 | -1.58 | -1.96 | 1.16 | 1.03 |
| BV | | -1.63 | -1.42 | -0.17 | 1.09 |
| MinRV | | | -1.00 | 1.66 | 1.17 |
| MedRV | | | | 2.14 | 1.16 |
| $TRV_6$ | | | | | 0.98 |

Table 1.4: GW test t-statistics for BAC (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.26 | -1.33 | 0.32 | 1.02 | 1.32 |
| BV | | -1.15 | 1.15 | 1.07 | 1.31 |
| MinRV | | | 1.39 | 1.14 | 1.34 |
| MedRV | | | | 1.03 | 1.33 |
| $TRV_6$ | | | | | 1.55 |

Table 1.5: GW test t-statistics for GE (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 2.90 | -0.82 | -0.78 | 2.22 | 3.22 |
| BV | | -2.18 | -1.94 | 1.30 | 3.08 |
| MinRV | | | 0.69 | 1.68 | 3.19 |
| MedRV | | | | 2.00 | 3.28 |
| $TRV_6$ | | | | | 2.67 |

Table 1.6: GW test t-statistics for IBM (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 0.81 | 0.54 | 1.16 | 2.63 | 2.46 |
| BV | | –1.87 | -0.93 | 0.95 | 1.98 |
| MinRV | | | 0.69 | 1.85 | 2.42 |
| MedRV | | | | 1.65 | 2.02 |
| $TRV_6$ | | | | | 1.39 |

Table 1.7: GW test t-statistics for JNJ (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 0.92 | 0.71 | -0.32 | 1.17 | 1.21 |
| BV | | -0.77 | -3.83 | 0.66 | 0.94 |
| MinRV | | | -1.35 | 2.04 | 1.94 |
| MedRV | | | | 1.84 | 1.71 |
| $TRV_6$ | | | | | 1.29 |

Table 1.8: GW test t-statistics for MO (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 0.70 | 1.45 | 1.54 | 2.28 | 2.64 |
| BV | | -2.45 | -2.73 | 1.50 | 2.77 |
| MinRV | | | 0.36 | 5.50 | 5.43 |
| MedRV | | | | 5.73 | 5.10 |
| $TRV_6$ | | | | | 2.99 |

Table 1.9: GW test t-statistics for PFE (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 0.64 | 1.41 | 1.72 | 2.95 | 2.10 |
| BV | | -0.73 | -0.62 | -0.04 | 1.28 |
| MinRV | | | 0.54 | 0.40 | 1.89 |
| MedRV | | | | 0.33 | 1.88 |
| $TRV_6$ | | | | | 1.28 |

Table 1.10: GW test t-statistics for PG (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.69 | 0.96 | 0.12 | 1.41 | 1.79 |
| BV | | 0.24 | -1.15 | 1.16 | 1.57 |
| MinRV | | | -1.28 | 1.82 | 1.35 |
| MedRV | | | | 1.51 | 1.45 |
| $TRV_6$ | | | | | 1.12 |

Table 1.11: GW test t-statistics for T (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 0.99 | -0.02 | 0.03 | 1.19 | 1.56 |
| BV | | -1.01 | -0.86 | 0.05 | 1.28 |
| MinRV | | | 0.37 | 0.63 | 1.58 |
| MedRV | | | | 0.56 | 1.49 |
| $TRV_6$ | | | | | 1.03 |

Table 1.12: GW test t-statistics for WMT (Large cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.99 | 0.44 | -0.46 | 1.45 | 2.12 |
| BV | | -5.61 | -9.48 | -2.30 | 0.70 |
| MinRV | | | -5.11 | 4.16 | 4.07 |
| MedRV | | | | 5.39 | 4.42 |
| $TRV_6$ | | | | | 3.32 |

Table 1.13: GW test t-statistics for AKS (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 2.10 | 1.10 | 1.19 | 1.60 | 1.98 |
| BV | | -0.76 | -0.57 | 1.22 | 2.10 |
| MinRV | | | 0.24 | 1.24 | 2.28 |
| MedRV | | | | 1.89 | 1.98 |
| $TRV_6$ | | | | | 1.43 |

Table 1.14: GW test t-statistics for ATW (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 2.20 | 3.38 | 4.02 | 4.80 | 3.31 |
| BV | | -0.11 | 1.03 | -2.12 | 0.93 |
| MinRV | | | 1.44 | -1.49 | 0.90 |
| MedRV | | | | -2.18 | 0.51 |
| $TRV_6$ | | | | | 1.94 |

Table 1.15: GW test t-statistics for BPL (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.67 | 1.97 | 2.28 | 1.47 | 1.78 |
| BV | | 1.29 | -0.70 | 0.22 | 1.28 |
| MinRV | | | -1.06 | -0.58 | 1.21 |
| MedRV | | | | 0.49 | 1.28 |
| $TRV_6$ | | | | | 1.10 |

Table 1.16: GW test t-statistics for CTV (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 3.19 | 3.64 | 4.21 | 5.01 | 4.94 |
| BV | | 0.66 | 0.61 | -1.20 | 1.86 |
| MinRV | | | 0.10 | -1.25 | 1.46 |
| MedRV | | | | -1.47 | 1.48 |
| $TRV_6$ | | | | | 3.45 |

Table 1.17: GW test t-statistics for CW (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.73 | 1.07 | 1.39 | 1.37 | 1.83 |
| BV | | -1.02 | 1.55 | 0.87 | 2.05 |
| MinRV | | | 1.40 | 1.18 | 1.86 |
| MedRV | | | | -0.41 | 2.03 |
| $TRV_6$ | | | | | 2.31 |

Table 1.18: GW test t-statistics for DRQ (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 3.93 | 4.63 | 5.68 | 4.53 | 4.00 |
| BV | | -1.21 | -1.48 | -1.56 | 1.39 |
| MinRV | | | -0.82 | -0.58 | 1.76 |
| MedRV | | | | -0.13 | 2.05 |
| $TRV_6$ | | | | | 2.02 |

Table 1.19: GW test t-statistics for HXL (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 4.78 | 4.52 | 4.96 | 4.91 | 4.53 |
| BV | | -1.57 | -1.82 | -1.87 | 0.45 |
| MinRV | | | -1.50 | -0.51 | 1.13 |
| MedRV | | | | -0.04 | 1.42 |
| $TRV_6$ | | | | | 2.48 |

Table 1.20: GW test t-statistics for KEX (Mid cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 3.79 | 4.37 | 5.20 | 5.70 | 4.57 |
| BV | | -0.49 | 0.61 | 0.25 | 2.43 |
| MinRV | | | 1.17 | 0.48 | 2.63 |
| MedRV | | | | -0.24 | 2.69 |
| $TRV_6$ | | | | | 2.55 |

Table 1.21: GW test t-statistics for RBA (Mid Cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 3.70 | 5.41 | 5.64 | 4.27 | 5.45 |
| BV | | -0.11 | -1.69 | -2.72 | 2.54 |
| MinRV | | | -1.33 | -2.13 | 2.38 |
| MedRV | | | | -1.92 | 2.71 |
| $TRV_6$ | | | | | 3.84 |

Table 1.22: GW test t-statistics for VMI (Mid Cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 2.82 | 2.31 | 2.17 | 2.83 | 2.94 |
| BV | | 0.10 | -0.70 | 0.03 | 2.11 |
| MinRV | | | 0.74 | -0.04 | 2.08 |
| MedRV | | | | 0.52 | 1.92 |
| $TRV_6$ | | | | | 2.10 |

Table 1.23: GW test t-statistics for CCC (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 10.06 | 8.69 | 9.94 | 8.26 | 9.35 |
| BV | | -0.39 | -0.31 | -5.78 | 1.25 |
| MinRV | | | 0.09 | -4.28 | 1.13 |
| MedRV | | | | -5.90 | 1.25 |
| $TRV_6$ | | | | | 5.75 |

Table 1.24: GW test t-statistics for CMO (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.30 | 1.38 | 1.95 | 3.02 | 3.04 |
| BV | | -0.60 | -0.42 | -0.18 | 2.29 |
| MinRV | | | 0.11 | 0.32 | 2.18 |
| MedRV | | | | 0.42 | 2.25 |
| $TRV_6$ | | | | | 2.26 |

Table 1.25: GW test t-statistics for CIA (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 3.06 | 2.91 | 2.91 | 2.74 | 3.15 |
| BV | | -3.00 | -2.31 | -2.96 | -0.17 |
| MinRV | | | -0.63 | -2.02 | 0.83 |
| MedRV | | | | -2.23 | 1.15 |
| $TRV_6$ | | | | | 4.05 |

Table 1.26: GW test t-statistics for CRN (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 6.19 | 6.09 | 3.62 | 5.83 | 1.67 |
| BV | | 0.82 | 0.92 | 0.04 | 1.02 |
| MinRV | | | 0.96 | -1.96 | 1.04 |
| MedRV | | | | -1.23 | 1.06 |
| $TRV_6$ | | | | | 1.10 |

Table 1.27: GW test t-statistics for CV (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 6.05 | 5.82 | 5.99 | 5.67 | 5.70 |
| BV | | 0.77 | 1.29 | 0.16 | 0.38 |
| MinRV | | | 2.29 | -0.85 | -0.41 |
| MedRV | | | | -2.39 | -1.58 |
| $TRV_6$ | | | | | 0.58 |

Table 1.28: GW test t-statistics for CYD (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.55 | 1.51 | 1.51 | 1.60 | 1.57 |
| BV | | -1.51 | -1.66 | -0.06 | 1.57 |
| MinRV | | | -1.80 | 0.34 | 1.63 |
| MedRV | | | | 0.51 | 1.73 |
| $TRV_6$ | | | | | 1.11 |

Table 1.29: GW test t-statistics for DCO (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 6.83 | 6.51 | 7.47 | 8.16 | 7.37 |
| BV | | -2.81 | 0.01 | -1.14 | 0.06 |
| MinRV | | | 1.94 | -0.63 | 0.64 |
| MedRV | | | | -1.30 | 0.06 |
| $TRV_6$ | | | | | 1.63 |

Table 1.30: GW test t-statistics for HGR (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 1.14 | 1.15 | 1.15 | 1.10 | 1.13 |
| BV | | 1.05 | 0.59 | -1.49 | -0.89 |
| MinRV | | | 0.01 | -1.70 | -1.26 |
| MedRV | | | | -2.03 | -1.17 |
| $TRV_6$ | | | | | 1.41 |

Table 1.31: GW test t-statistics for MIG (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | 6.83 | 7.08 | 7.27 | 6.73 | 6.99 |
| BV | | -0.92 | -2.12 | -1.90 | 1.47 |
| MinRV | | | -1.76 | -1.42 | 1.71 |
| MedRV | | | | -0.91 | 2.19 |
| $TRV_6$ | | | | | 2.32 |

Table 1.32: GW test t-statistics for RHB (Low cap) using MSE distance measure (2min frequency). A positive t-stat means that the column variable outperforms the row variable.

| 5min MedRV as proxy | | | | | |
|---|---|---|---|---|---|
| | BV | MinRV | MedRV | $TRV_6$ | $TRV_3$ |
| RV | -0.16 | 3.05 | 1.99 | 4.29 | 3.30 |
| BV | | -0.68 | -89 | -0.79 | 2.00 |
| MinRV | | | -0.74 | -0.46 | -1.86 |
| MedRV | | | | 0.60 | 1.37 |
| $TRV_6$ | | | | | 1.77 |

Table 1.33: Model Confidence Set (Hansen, Lunde, and Nason (2011)) Results for 10 large cap stocks. The table reports the number of times that the estimator was found to be one of the best models (in the model confidence set).

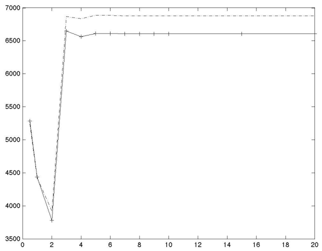
| Frequency | RV | BV | MinRV | MedRV | $TRV_6$ | Truncation level | $TRV_{2min}$ |
|---|---|---|---|---|---|---|---|
| 10s | 2 | 4 | 3 | 3 | 3 | 1 | 3 |
| 30s | 2 | 6 | 3 | 3 | 4 | 2 | 9 |
| 1m | 2 | 8 | 3 | 3 | 5 | 3 | 9 |
| 2m | 2 | 8 | 6 | 3 | 8 | 4 | 9 |
| 5m | 3 | 5 | 6 | 6 | 9 | 5 | 9 |
| 10m | 3 | 7 | 6 | 7 | 7 | | |
| 15m | 3 | 6 | 7 | 6 | 6 | | |
| 20m | 3 | 6 | 4 | 6 | 6 | | |
| 30m | 3 | 4 | 4 | 5 | 5 | | |

Table 1.34: Model Confidence Set (Hansen, Lunde, and Nason (2011)) Results for 10 medium cap stocks. The table reports the number of times that the estimator was found to be one of the best models (in the model confidence set).

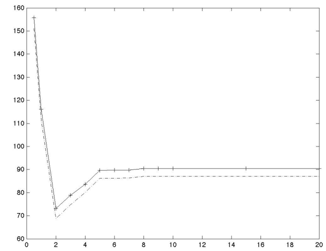
| Frequency | RV | BV | MinRV | MedRV | $TRV_6$ | Truncation level | $TRV_{2min}$ |
|---|---|---|---|---|---|---|---|
| 10s | 2 | 6 | 6 | 8 | 8 | 1 | 4 |
| 30s | 2 | 9 | 9 | 9 | 9 | 2 | 10 |
| 1m | 2 | 8 | 7 | 9 | 8 | 3 | 10 |
| 2m | 2 | 8 | 8 | 8 | 5 | 4 | 8 |
| 5m | 3 | 7 | 8 | 6 | 4 | 5 | 5 |
| 10m | 3 | 6 | 5 | 6 | 4 | | |
| 15m | 2 | 5 | 2 | 3 | 3 | | |
| 20m | 2 | 5 | 2 | 3 | 3 | | |
| 30m | 2 | 4 | 3 | 4 | 3 | | |

Table 1.35: Model Confidence Set (Hansen, Lunde, and Nason (2011)) Results for 10 small cap stocks. The tables reports the number of times that the estimator was found to be one of the best models (in the model confidence set).

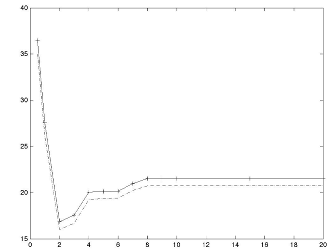
| Frequency | RV | BV | MinRV | MedRV | $TRV_6$ | Truncation level | $TRV_{2min}$ |
|---|---|---|---|---|---|---|---|
| 10s | 3 | 4 | 4 | 5 | 8 | 1 | 5 |
| 30s | 3 | 4 | 5 | 7 | 8 | 2 | 10 |
| 1m | 3 | 8 | 7 | 8 | 6 | 3 | 9 |
| 2m | 3 | 8 | 8 | 7 | 3 | 4 | 5 |
| 5m | 3 | 6 | 4 | 5 | 3 | 5 | 3 |
| 10m | 3 | 4 | 4 | 4 | | | |
| 15m | 3 | 3 | 3 | 3 | 3 | | |
| 20m | 3 | 3 | 3 | 3 | 3 | | |
| 30m | 3 | 3 | 3 | 3 | 3 | | |



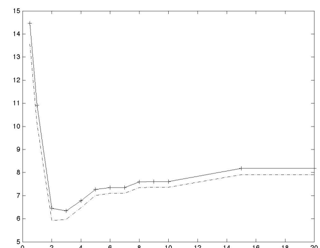
(a) AIG



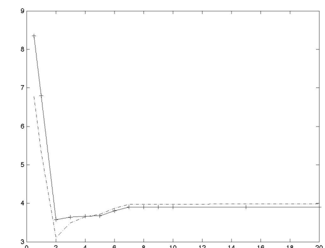
(b) BAC



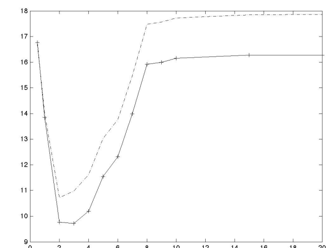
(c) GE



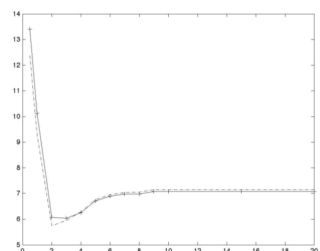
(d) IBM



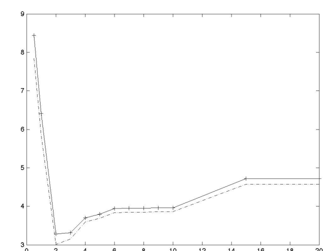
(e) JNJ



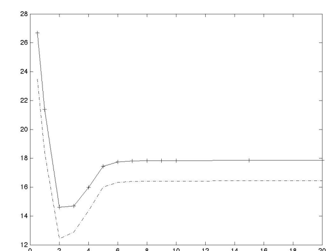
(f) MO



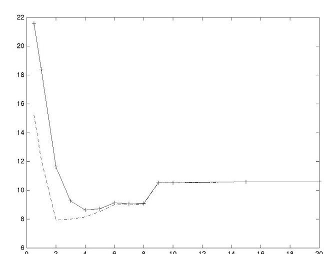
(g) PFO



(h) PG

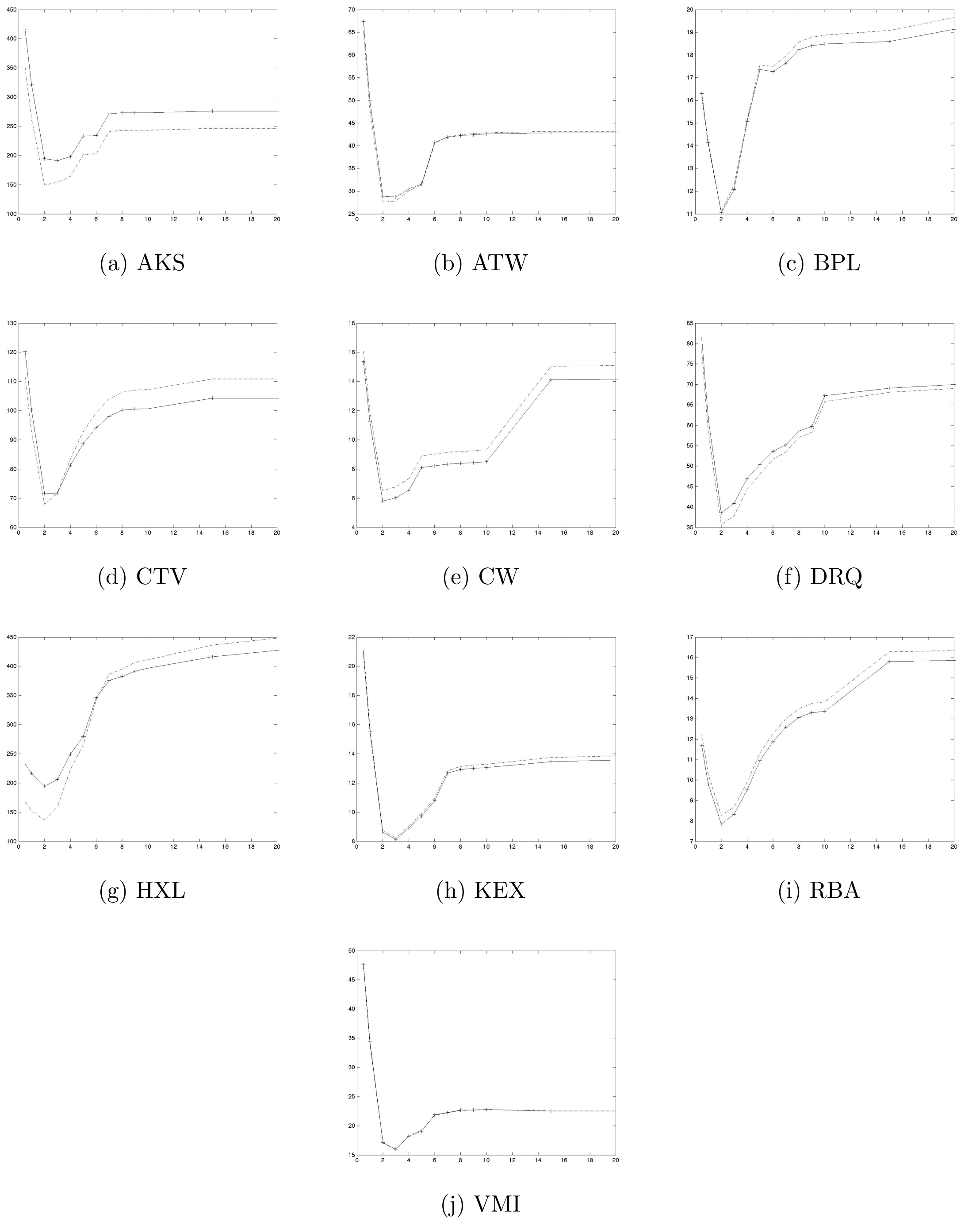


(i) T



(j) WMT

FIGURE 1.1: Average MSE loss for $\mathrm{TRV}_{2min}$ where the truncation parameter varies along the x-axis. Large cap stocks. The cross is the 2 minute MedRV proxy and the dashed line is the 2 minute BV (bipower variation) proxy.



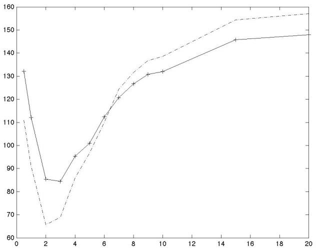
(a) AKS (b) ATW (c) BPL

(d) CTV (e) CW (f) DRQ

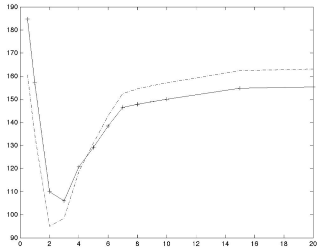
(g) HXL (h) KEX (i) RBA

(j) VMI

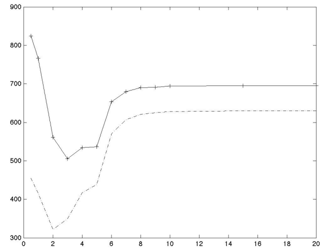
FIGURE 1.2: Average MSE loss for $TRV_{2min}$ where the truncation parameter varies along the x-axis. Mid cap stocks.The cross is the 2 minute MedRV proxy and the dashed line is the 2 minute BV (bipower variation) proxy.



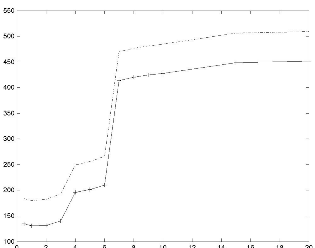
(a) CCC



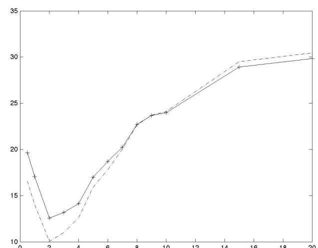
(b) CIA



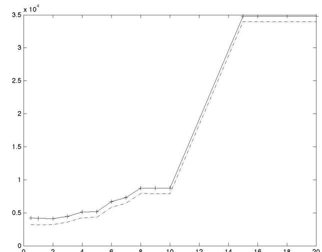
(c) CMO



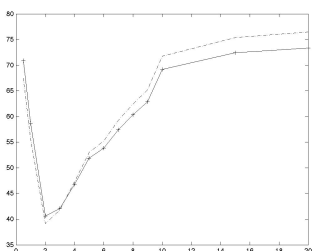
(d) CRN



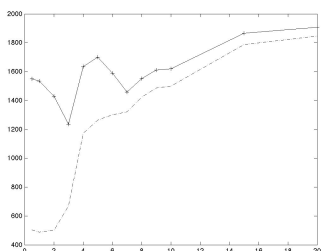
(e) CV



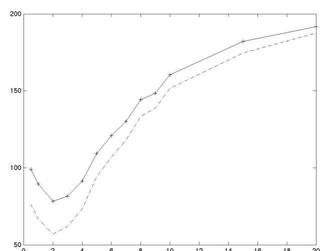
(f) CYD



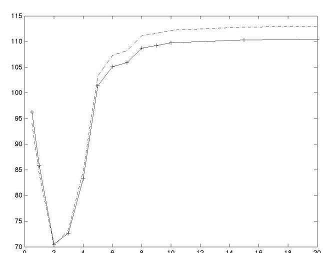
(g) DCO



(h) HGR



(i) MIG



(j) RHB

FIGURE 1.3: Average MSE loss for $\text{TRV}_{2min}$ where the truncation parameter varies along the x-axis. Small cap stocks. The cross is the 2 minute MedRV proxy and the dashed line is the 2 minute BV (bipower variation) proxy.

# 2

# Examining the Commonality in Liquidity and Volatility Risk

## 2.1 Introduction

Recently, there have been two separate paths that explore the cross-section of stock returns. One emphasizes the importance of volatility as a systematic risk factor (e.g. Ang et al. (2006), Adrian and Rosenberg (2008) and Moise (2007)), while the other focuses on systematic liquidity risk (see Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Chen (2005), and Sadka (2006)). Additionally, since there are several different measures of liquidity, several studies have focused on identifying a common systematic liquidity factor (see Chordia et al. (2000), Hasbrouck and Seppi (2001), and Eckbo and Norli (2002)). While much work has been done focusing on liquidity and volatility separately, relatively little work has been done exploring the joint pricing of systematic liquidity and volatility risk.

Liquidity and volatility arise from differing economic causes, with volatility resulting from fluctuations in asset valuations and liquidity caused by market trading frictions. However, it is possible that they are both proxies for another more fundamental factor, which we will refer to as "uncertainty," which varies with the state of the economy. If this is the case, it would be interesting to see if the explanatory power of either liquidity or volatility risk is reduced in a joint asset pricing model. Bandi et al. (2008) examine this question at the market level using measures of market liquidity and volatility risk derived from high frequency prices of the SPDR (a trust invested in the S&P 500). They find that when considering liquidity or volatility risk individually they are significant risk factors, however, in the model which includes both liquidity and volatility risk only the volatility risk is significant. They conclude that this likely results because they each are proxies for a more fundamental underlying factor. This paper will further explore whether a common uncertainty factor derived from liquidity and volatility risk is significantly priced in the cross-section and if this may drive the results when volatility and liquidity risk are considered separately.

In order to better understand the disparate liquidity measures, Korajczyk and Sadka (2008) examine eight different measures of liquidity to determine whether they are each capturing a common underlying liquidity factor or whether there are potentially multiple liquidity risk factors each captured by a different measure. Using the technique of Connor and Korajczyk (1987), Korajczyk and Sadka (2008) are able to extract an across-measure liquidity factor derived from the stacking of all individual liquidity measures. They find the across-measure liquidity factor is what is significant in the cross section, not anything unique to the various measures.

Using analysis techniques similar to those in Korajczyk and Sadka (2008), we extract latent factors for multiple liquidity and volatility measures on a sample of 4975 NYSE stocks over the period of July 1962 to December 2011. In addition to risk factors specific to each individual measure, across-measure liquidity (volatility) factors are estimated by considering multiple liquidity (volatility) measures. Common, what we will term as "uncertainty," factors are extracted across all of the liquidity and volatility measures. We use these various factors to further examine the joint pricing of liquidity and volatility risk in the cross-section.

One liquidity measure included in the study is the Amihud (2002) measure, the ratio of absolute returns and dollar volume. This is a gauge of price impact since it measures the daily price response for each dollar of trading. Also considered is the relative spread, the ratio of the bid-ask spread and the midpoint price (see Næs et al. (2011)), the Roll (1984) measure which is based on the autocorrelation of daily returns, and the turnover, the ratio of volume to shares outstanding. The volatility measures are monthly realized variance (sum of squared returns), a monthly measure using the open, close, high, and low prices (see Garman and Klass (1980)), and monthly estimates from a GARCH(1,1) specification.

Our results indicate that there does exist a fundamental uncertainty factor that is related to both systematic liquidity and volatility as well as returns. Pair-wise canonical correlations show that shocks to liquidity and volatility are correlated to the common uncertainty factor and contemporaneously correlated to returns. Liquidity factors are highly persistent while volatility factors exhibit a lower degree of persistence. The shocks to liquidity and volatility factors are estimated as the residuals of an AR(2) model.

The final analysis examines the cross-sectional pricing of liquidity risk, volatility risk, and the common uncertainty risk in addition to the premium on the raw liquidity and volatility levels. The across-measure liquidity and volatility factors are orthogonalized to the common uncertainty factor to better isolate the risk specific to liquidity and volatility. We find that uncertainty risk is significantly priced in the cross-section while the risk attributed solely to liquidity and volatility is not. This suggests that liquidity and volatility risk are both (weak) proxies for an underlying risk factor, we choose to call this uncertainty risk, which drives the significant pricing results when considering liquidity and volatility individually.

The paper is organized as follows. Section 2 discusses the specific liquidity and volatility measures as well as the method for extracting the risk factors. Section 3 presents the AR(2) results and explores both the pair-wise contemporaneous and lead-lag correlations of the risk factors and returns. Section 4 presents the cross-sectional pricing analysis and Section 5 concludes.

## 2.2 Data

This paper utilizes data from the daily and monthly CRSP databases for stocks traded on the NYSE between July 1962 to December 2011. Since trading on the NASDAQ uses a different trading mechanism relying heavily on market makers, only stocks traded on the NYSE are considered in the analysis. Additionally, only assets with a CRSP share code of 10 or 11 (ordinary common shares) are considered which will eliminate certificates, Americus Trusts components, ADRs, shares of beneficial interest, closed-end funds, REIT's, and ETFs. Stocks with a price lower than \$1 are excluded as well as those observations with a volume = 0. After appropriate filtering, we are left with a total of 4975 firms over a total of 594 months.

### *2.2.1 Liquidity Measures*

There are a wide range of proposed measures of liquidity. We implement a total of four liquidity measures. The first is the measure based on Amihud (2002). Define the Amihud measure for stock $i$ in month $t$ as

$$A_{i,t} = \frac{1}{d_t} \sum_{j=1}^{d_t} \frac{|r_{i,j}|}{dvol_{i,j}} \tag{2.1}$$

where $r_{i,j}$ is the return on asset $i$ on day $j$ of month $t$, $d_t$ is the number of trading days in the month, and $dvol_{i,j}$ is the dollar volume for asset $i$ on day $j$ of month $t$. Following both Acharya and Pedersen (2005) and Korajczyk and Sadka (2008), the monthly measure $A_{i,t}$ is scaled by the ratio of the market capitalization of the CRSP market index at time $t-1$ and at the reference date of July 1962. In order for the monthly measure to be included in the sample, a stock is required to have at least 15 daily observations.

The second liquidity measure employed is the turnover, the ratio of monthly volume and shares outstanding. It is defined as

$$TO_{i,t} = \frac{\sum_{j=1}^{d_t} vol_{i,j}}{SO_{i,t}} \tag{2.2}$$

where $SO_{i,t}$ is the number of shares outstanding at the end of month $t$. Once again, it is required that a stock have at least 15 daily observations in month $t$ to be included in the sample.

The relative spread is calculated as the difference between the bid and the ask divided by the midpoint price (average of the bid and ask).

$$RS_{i,t} = \frac{1}{d_t} \sum_{j=1}^{d_t} \frac{Ask_{i,j} - Bid_{i,j}}{midpt_{i,j}} \tag{2.3}$$

This is calculated at the daily frequency and then aggregated by taking the monthly average of the daily measures. The purpose of the relative spread is to measure the implicit cost of trading a small number of shares.

The final liquidity measure employed is that of Roll (1984). Assuming the existence of a constant spread $s$, Roll shows that the spread can be estimated as $\hat{s} = 2\sqrt{-Scov}$ where $Scov$ is the covariance of adjacent daily returns. This is estimated each month using daily returns where a minimum of 15 daily returns is required to be included. Since this is undefined when $Scov > 0$, those liquidity estimates are set to missing.[1]

[1] Harris (1990) suggests using $\hat{s} = -2\sqrt{Scov}$ when $Scov < 0$, but this would result in a negative spread which would imply a negative transaction cost. Since this isn't meaningful, months with $Scov > 0$ are simply set to missing as in Næs et al. (2011).

### *2.2.2 Volatility Measures*

Three different estimates of monthly volatility are employed in the following analysis. The first is an estimate formed from the daily realized variance measure simply defined as

$$RV_{i,t} = \sum_{j=1}^{d_t} r_{i,j}^2 \tag{2.4}$$

where, again, $r_{i,j}$ is the return of asset $i$ on day $j$ of month $t$ and $d_t$ is the number of trading days in month $t$.

Garman and Klass (1980) find that the best analytic scale-invariant estimator of daily volatility, $\sigma_j^2$, is

$$GK_{i,t} = 0.51(u_j - d_j)^2 - 0.019[c_j(u_j + d_j) - 2u_j d_j] - 0.383c_j^2 \tag{2.5}$$

where $c_j$ is the closing cost, $u_j$ is the daily high, and $d_j$ is the low. Each of the terms is normalized by subtracting the daily opening price. Once the daily estimates are calculated, the monthly estimate is obtained by summing the daily estimate over the days of the month.

The final estimate of monthly volatility for each asset is obtained by estimating a simple GARCH(1,1) model over an expanding window with a minimum of 24 monthly returns required for estimation. Formally, the monthly variance for our GARCH(1,1) model is defined as

$$r_t = c + \epsilon_t \qquad \epsilon \sim N(0, \sigma_t^2) \tag{2.6a}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_i \sigma_{t-1}^2 \tag{2.6b}$$

In order to reduce the effects of outliers, each estimate of liquidity and volatility is Windsorized at the 1st and 99th cross-sectional percentiles for each month[2].

[2] To illustrate, consider the variance estimate $RV_{i,t}$. Let $RV_t^{99\%}$ be the 99th percentile of all RV estimates for the month $t$. If $RV_{j,t} > RV_t^{99\%}$ then $RV_{j,t}$ is set equal to $RV_t^{99\%}$. Similarly, any monthly measure that is less than $RV_t^{1\%}$ will be set equal to the 1st percentile.

This results in an unbalanced panel of 4 liquidity and 3 volatility measures over 4975 NYSE firms spanning a total of 594 months. The various liquidity (volatility) measures will be used to derive a common liquidity (volatility) factor. A common cross-sectional factor will be extracted from the combined liquidity and volatility measures which we will refer to as the common uncertainty factor.

### *2.2.3 Factor decomposition*

We will be examining the common uncertainty factor across the various liquidity and volatility measures using a process similar to that of Korajczyk and Sadka (2008). Since the units are not comparable for the various liquidity and volatility measures, each measure is standardized using the mean and standard deviation in the cross section using all available data prior to month $t$. Specifically, let $M^i$ be the $n \times T$ matrix of estimator $i$ (this could be either a liquidity or a volatility estimator). Define $\hat{\mu}^i_{t-1}$ and $\hat{\sigma}^i_{t-1}$ as the cross-sectional mean and standard deviation for measure $i$ estimated for all of the sample up to $t-1$. Then the standardized measure is calculated as $S^i_{j,t} = (M^i_{j,t} - \hat{\mu}^i_{t-1})/\hat{\sigma}^i_{t-1}$. The estimator $S^i$ is assumed to follow the factor model

$$S^i = B^i F^i + \epsilon^i, \tag{2.7}$$

where $F^i$ is a $k \times T$ matrix of shocks to the liquidity (volatility) measure that are common across the set of $n$ assets, $B^i$ is a $n \times k$ matrix of sensitivities to the common factor, and $\epsilon^i$ is the $n \times T$ matrix of asset specific shocks to the liquidity (volatility) measure. Connor and Korajczyk (1986) show that $n$-consistent estimates of the factors, $F^i$, are obtained by calculating the eigenvalues of

$$\Omega^i = \frac{S^{i\prime} S^i}{n}. \tag{2.8}$$

While this estimator relies on a balanced panel, it does vastly simplify the calculations as we are now simply calculating the eigenvectors of a $T \times T$ matrix which is independent of the number of stocks in our sample. In order to accommodate the fact that our panel is unbalanced, we follow the estimation technique of Connor and Korajczyk (1987) which will essentially estimate the elements of $\Omega$ using only the observed data. In order to implement this method, all of the missing observations in $S^i$ are replaced with zeros and the resulting balanced panel will be called $S^{i*}$. Define $N^i$ as a $n \times T$ indicator matrix where each element takes a value of 1 if the element in $S^i$ is observed or 0 if the corresponding element in $S^i$ is missing. Now we can construct an unbalanced equivalent of $\Omega$ that only uses the cross-sectional averages of the observed data.

$$\Omega^{i,u}_{t,\tau} = \frac{(S^{i*\prime} S^{i*})_{t,\tau}}{(N^{i\prime} N^i)_{t,\tau}} \tag{2.9}$$

The estimates of the $k$ latent factors, $\hat{F}^i$, can be calculated as the eigenvectors $(T \times 1)$ of the $k$ largest eigenvalues of $\Omega^{i,u}$. Following Connor and Korajczyk (1986), the eigenvectors are normalized so that the rows have a mean-square of 1.

For each measure, including returns, the first three principal components are extracted. A time series regression of each stock's measure on its common factor was performed in order to test the degree of commonality across assets. The resulting average $R^2$ values for regressions involving one, two, and three factors are reported in Table 2.1.

These cross-sectional results demonstrate that there is a high degree of commonality within each of the liquidity and volatility measures with the cross-sectional average $R^2$ ranging from 16.1% to 48.6% for one factor and increasing to a range of 35.8% to 69.9% with the inclusion of all three factors. There is little difference between the $R^2$ values for the liquidity and volatility measures. For stock returns, increasing the model from one to three factors results in a much more modest gain in average $R^2$ than for the liquidity and volatility measures. These results are in line with those of Chordia et al. (2000) who document a commonality among quoted and effective spread using data from 1992. They are also consistent with those of Korajczyk and Sadka (2008) who find a similar degree of commonality across various liquidity measures.

In addition to estimating the cross-sectional factors within each measure, common factors across all of the liquidity (volatility) measures are extracted as well. This can be accomplished by stacking the multiple liquidity (volatility) measures and then using the stacked matrix to form $\Omega$. The factors extracted from the stacked liquidity (volatility) measures will be referred to as the common, or across-measure, liquidity (volatility) factors. The sign of the liquidity factors is chosen so that an increase in the factor will correspond to an increase in liquidity. This is done by choosing the sign so that the within-measure factors are negatively correlated with the cross-sectional mean of the measure (although for turnover it will be positively correlated).

To better understand whether liquidity and volatility measures may simply be weak proxies of an underlying uncertainty measure, we stack all of the liquidity and volatility measure to extract what we call the common uncertainty factors.

## 2.3 Correlation analysis

### *2.3.1 Time series properties*

The autocorrelation function of the first factor for each liquidity and volatility measure, including a two standard deviation band, are plotted in Figure 2.1. In order to separate the factors into expected changes and unexpected shocks, an AR(2) is fit to each factor series. Calculating the residuals from the AR(2) regression will yield an estimate of the factor shocks. This is similar to Pástor and Stambaugh (2003) and Acharya and Pedersen (2005) who calculate shocks to liquidity using the residuals to an AR(2) process. The resulting AR(2) estimates are presented in Table 2.2.

As a measure of the persistence of each factor, the impulse response measured at time $t + 12$ to a shock at time $t$ is presented with the AR(2) estimates. The liquidity factors tend to be more persistent than the volatility factors, although both across-measure factors exhibit a degree of persistence. Returns, however, show very little persistence.

### *2.3.2 Contemporaneous canonical correlations*

The pairwise canonical correlations for the liquidity and volatility factors are calculated using the first three factors for each measure across pairs of measures. This will calculate the maximum correlation between linear combinations of the first three factors for any two measures[3]. The results for the raw factors are presented in Table 2.3 while Table 2.4 contains the results for the pre-whitened factors, or factor shocks obtained as the residuals from an AR(2) model.

The correlations for the raw measures are slightly higher than those of the pre-whitened factors. In almost all cases, they are highly correlated especially within the liquidity and volatility groups specifically, although the Amihud measure tends to have a lower correlation with the other measures. The common liquidity and volatility factors are highly correlated with each other as well as the common factor. The individual measures tend to have a higher correlation with their respective "common" factor; for instance the correlation of the Amihud factor with the common (across-measure) liquidity factor is larger than its correlation with the "common" volatility factors. As a whole, Tables 2.3 and 2.4 suggest there are strong correlations across all liquidity and volatility factors with most also being highly correlated with the common uncertainty factor. Such large correlations suggest that there is a degree of commonality across the liquidity and volatility measures and that they are contemporaneously correlated with each other and with returns. This result is consistent with recent studies that suggest liquidity and volatility risks are priced

[3] For example, the very first value of Table 2.3, or 0.098, corresponds to the maximum correlation between a linear combination of the first 3 factors for the Amihud measure and the first three factors of the cross-sectional returns.

factors (see e.g. Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Sadka (2006), Ang et al. (2006), Adrian and Rosenberg (2008), and Moise (2007)).

### *2.3.3 Predictability of returns, liquidity, and volatility*

In the next sections we will examine the relationship between liquidity, volatility, and uncertainty on expected returns. Now, we will focus on the relationship between shocks to liquidity, volatility, and uncertainty and shocks to future returns. Similarly, we will examine whether shocks to returns affect future shocks to liquidity and volatility. To answer these questions, we examine the pair-wise lead-lag canonical correlations of the shocks to returns and the liquidity, volatility, and uncertainty factors. This is similar to the previous correlation analysis except that one of the factors will be lagged. The results for the raw factors are presented in Table 2.5 while the lead-lag correlations for the pre-whitened shocks are presented in Table 2.6.

The first column of both Table 2.5 and Table 2.6 indicate that there is a weak relation between lagged liquidity and volatility factors returns. The relation between lagged returns and the liquidity and volatility measures is stronger and suggests that shocks to returns are able to predict shocks to liquidity and volatility. Similarly, lagged liquidity and volatility shocks have a potential for predicting shocks to our aggregate uncertainty factor.

Tables 2.5 and 2.6 examine the relation strictly focusing on a one month lag. However, predictability may not be restricted to one month but could extend beyond that horizon. Figure 2.2 displays the pairwise canonical lead-lag correlations using the first three factors of each measure. To better understand Figure 2.2, let's take a closer look at the plot in the upper left corner. For lag 0, this plot shows the contemporaneous canonical correlation for the shocks of the Amihud factors and the shocks to the returns factors. At lag 4, the plot takes the value of the canonical correlation between the return factors at time $t$ and the Amihud factors at time

$t + 4$. The slight spike at lag 4 would imply that shocks to returns at time $t$ are correlated with shocks to the Amihud factor four months ahead, at time $t + 4$. The resulting conclusion from the correlations between liquidity and volatility shocks and returns is that liquidity and volatility can be predicted by returns, but the opposite does not appear to hold as the correlations between lagged liquidity (volatility) and returns is much weaker. We now examine the relation of liquidity, volatility, and uncertainty and expected returns.

## 2.4 Joint pricing of liquidity and volatility risk in the cross-section

### *2.4.1 Portfolio construction and testing*

In this section we examine whether liquidity and volatility risk, or their absolute levels, are jointly priced in the cross section. As has been noted above, several papers (e.g. Ang et al. (2006), Adrian and Rosenberg (2008) and Moise (2007)) have found that volatility risk is priced in the cross-section while others (see Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Chen (2005), and Sadka (2006)) have found a similar result for liquidity risk. These papers consider either liquidity or volatility risk separately while Bandi et al. (2008) examines the joint pricing of liquidity and volatility at the market level. They find that when accounting for both risk factors, only volatility is a significant risk factor. They conclude with the thought that liquidity and volatility may both be weak proxies for an underlying uncertainty measure which could explain why only volatility is significant in the joint analysis while both are significant when examined individually.

The first step is to orthogonalize the liquidity and volatility factors from the common uncertainty factor. Let $\hat{F}_t^{LIQ}$ denote the common liquidity factor with $\hat{F}_t^{VOL}$ as the first across-measure factor of volatility. The first across-measure common uncertainty factor (obtained by taking the first eigenvector of the stacked matrix of liquidity and volatility measures), is denoted as $\hat{F}_t^{U}$. Specifically, the liquidity and volatility factors are orthogonalized using the regression

$$\hat{F}_t^j = b_o^j + b_1^j \hat{F}_t^U + \hat{u}_t^j \tag{2.10}$$

where $j = \{LIQ, VOL\}$ and $\hat{u}^j$ is the orthogonalized liquidity (volatility) factor. All of the factors are first pre-whitened using the previous AR(2) specification.

The individual liquidity and volatility measures are then regressed on the common uncertainty factor as well as the across-measure liquidity (volatility) factor (depending on the group to which it belongs) and the measure specific factor. Both the common liquidity and volatility factors were orthogonalized with the common uncertainty factor. The percentage of firms with significant results, including a test for joint significance, are presented in Table 2.7. This table represents the relative importance of the different factors in explaining the variation in the firm specific liquidity and volatility measures. As is shown in Table 2.7, each of the factors is significant at a frequency higher than the test size. Also, for the majority of liquidity and volatility measures, the common uncertainty factor is statistically significant for over 20% of the firms (at the 5% level). The firm-specific volatility measures are impacted by the common uncertainty factor at a higher frequency than the liquidity measures.

To construct our portfolios for the cross-sectional analysis, we first estimate the systematic uncertainty risk using a factor model that includes the three Fama and French (1993) factors (excess market returns (MKT), high-minus-low (HML) book-to-market, and the small-minus-big (SMB) portfolio) and the momentum (UMD) of Carhart (1997).[4] We will collectively refer to these four factors as the FF4 factors. Factor betas are estimated in the first stage regression for each asset through the regression

$$R_{i,t} = \beta_{0,i} + \beta_i' f_t + \epsilon_{i,t} \tag{2.11}$$

[4] Thanks to Kenneth French for making these readily available on his website.

where $R_{i,t}$ is the excess return of asset $i$ and $f_t$ is a vector of factors. Each month, stocks are ranked according to their uncertainty risk, as measured by their beta on the common uncertainty factor using the previous 36 months. In order for a stock's beta to be estimated in month $t$, we require that there be at least 24 observations within the last 36 months. Based on this beta, the stock is assigned to one of 30 portfolios. Once the portfolios are constructed, the betas for the portfolios, which are assumed to be constant over the sample period, are estimated in the second stage regression using a similar factor model. This means that while the beta of a specific portfolio is assumed to be constant, stocks are allowed to move between portfolios as their specific betas may be changing. In this second stage regression, the orthoganalized liquidity and volatility factors are included in addition to the FF4 and the uncertainty factor.

Table 2.8 reports the portfolio's average monthly excess returns, Jensen's $\alpha$ for the factor pricing model using only the FF4 factors, and the post-ranking betas for the liquidity, volatility, and uncertainty factors. The t-statistics are calculated using the standard error adjustment of Newey and West (1987) with 5 lags. The loadings (betas) on the orthogonalized liquidity and volatility factors are significant for nearly all of the 30 portfolios while the betas on the uncertainty factor rarely exhibit statistical significance. Additionally, if liquidity, volatility, and uncertainty risk are not priced independently of the FF4 factors, there should be no relation between the portfolio $\alpha$'s and their betas. Regressing $\alpha$ on the betas yields the following estimates

$$\alpha_p = \underset{[11.73]}{0.014} - \underset{[-1.31]}{0.013}\beta_{LIQ} + \underset{[8.15]}{0.112}\beta_{VOL} - \underset{[-2.11]}{0.125}\beta_U \qquad R^2 = 0.58. \tag{2.12}$$

In the next section, we will test explicitly for pricing in the cross-section but these results suggest that the common uncertainty factor is significantly priced in the cross-section. Additionally, the volatility specific risk is significant in the cross-section, while liquidity risk may not be significantly priced in the cross-section.

### *2.4.2 Cross-sectional regressions*

The cross-sectional pricing models testing the pricing of liquidity and volatility risk are of the form

$$\mathrm{E}[R_i] = \lambda_0 + \lambda^{FF'} \beta_i^{FF} + \lambda' \beta_i \tag{2.13}$$

where $\mathrm{E}[R_i]$ is the expected return of portfolio $i$ in excess of the risk-free rate, $\beta^{FF}$ is the factor loadings for the FF4 factors and $\beta_i$ is the loadings for the liquidity, volatility, and uncertainty risk factors, and $\lambda^{FF}$ and $\lambda$ are vectors of the factor premia respectively. Specifically, the coefficients are estimated for each month $t = 1, 2, ..., T$ in the cross-sectional specification

$$R_{i,t} = \lambda_{0,t} + \lambda_t^{FF'} \beta_{i,t}^{FF} + \lambda_t' \beta_{i,t} + \nu_{i,t} \tag{2.14}$$

Equation (2.14) is estimated using the method of Fama and MacBeth (1973) where excess returns, $R_{i,t}$ are measured at the firm level. Since there can be much variability in firm specific betas, individual firms are assigned the betas associated with their uncertainty portfolio in month $t$. Similar to the previous subsection, firms are sorted into $m$ portfolios based on their exposure to the common uncertainty factor over the previous 36 months where a firm must have at least 24 observations in order to be included in a portfolio. Once the portfolios are constructed, the betas for each portfolio are assumed to be constant and are calculated over the entire sample. For month $t$, each firm is assigned the betas corresponding to its portfolio assignment. This procedure is in line with that of Fama and French (1992).

This cross-sectional estimation results in a time series of estimates, $\hat{\lambda}_t^{FF}$ and $\hat{\lambda}_t$. The time-series mean and standard deviation calculated with a Newey-West correction of 5 lags are presented in Table 2.9. The results in Table 2.9 support the conclusion that the significant pricing results can be attributed to the common underlying risk factor and this is consistent across a variety of portfolio sorts. Since the across-measure liquidity and volatility factors are orthogonalized with the common uncertainty factor, we are able to isolate the risk specific to liquidity and volatility. We find the common uncertainty risk factor, extracted across the pooled liquidity and volatility measures, is significantly priced in the cross-section. This implies that the liquidity and volatility may be proxying for an underlying (uncertainty) factor that has significant pricing in the cross-section which would explain the results in Bandi et al. (2008), who find the significance of liquidity risk vanishes once you jointly consider liquidity and volatility. The results are broadly consistent across a range in the number of sorted portfolios and show that the risk specific to liquidity and volatility are not priced, while their common underlying risk (which we have been calling uncertainty) is significantly priced in the cross section.

### *2.4.3 Liquidity and Volatility Risk*

In order to strengthen the argument that the significance of the underlying uncertainty risk factor is driving the significant results of liquidity and volatility risk, Equation (2.14) will be estimated using only the non-orthogonalized liquidity and volatility risk factors. Since we are not considering the common uncertainty risk factor, the portfolios will be performed by sorting the stocks on their exposure to liquidity or volatility risk.

As we can see in Table (2.10), liquidity risk is significantly priced in the cross section of returns when considered individually across a variety of portfolio sorts ranging from a 15 portfolios to 60 portfolios. The results are not as consistent for volatility risk, but it is weakly significant for a couple of portfolio sorts. Combining these results with the previous subsection, we find that the common underlying risk factor is driving the significance of the liquidity and volatility risk when considered individually. This further strengthens the argument that liquidity and volatility may be proxying for an underlying uncertainty risk.

## 2.5 Conclusion

Several studies find significant systematic liquidity and volatility risk when considered individually. However, liquidity and volatility risk are rarely considered jointly.[5] Bandi et al. (2008) examine liquidity and volatility risk jointly, but only at the market level and over a shorter sample (due to the reliance on high frequency data for their estimation). They find that both liquidity and volatility risk are significant when considered individually, but only volatility risk is significant in the joint specification. Their possible explanation is that liquidity and volatility are both proxies for a significant underlying uncertainty risk of which volatility is a better measure. This paper further examines the relationship between liquidity and volatility risk.

We calculate various liquidity and volatility measures across 4975 NYSE firms from July 1962 to December 2011. Latent factor models are estimated for each measure. Additionally, the latent factors of the pooled liquidity (volatility) measures are extracted to form across-measure liquidity (volatility) factors. To explore the possibility that they are both proxies for an underlying uncertainty factor, a latent factor model is estimated across the collection of both liquidity and volatility measures. We find that there is a high correlation between the common uncertainty factor and the individual liquidity and volatility measures. Shocks to returns are contemporaneously correlated to shocks to individual liquidity and volatility measures as well as shocks to the across-measure liquidity and volatility factors. Additionally, there is evidence that shocks to returns can predict shocks to both liquidity and volatility across assets. Liquidity shocks are very persistent while shocks to liquidity tend to have no impact after about 12 months.

For the cross-sectional pricing analysis, the across-measure liquidity and volatility

[5] Amihud (2002) does control for the raw annual volatility in his analysis of liquidity risk.

risk factors are orthogonalized from the uncertainty risk factor. Neither the liquidity specific risk factor nor the volatility specific risk factor exhibit significant pricing in the cross-section, while the common uncertainty risk is significant in the cross-sectional specification. However, when considered individually without extracting the common risk component, both liquidity and volatility risk are significant in the cross section of returns. These results indicate that both liquidity and volatility are proxies for an underlying and significant risk factor, which we term "uncertainty." Furthermore, the significant results in the liquidity and volatility literatures appear to result from the ability of the various liquidity and volatility measures to proxy for the underlying uncertainty risk.

## 2.6 Tables and Figures

Table 2.1: Degree of commonality in the measure specific factors.
This table reports the average $R^2$ for each stock's time series regression of its measure on the corresponding factors. A total of 3 factors were extracted for four liquidity, three volatility estimators, and the monthly returns using the method of Connor and Korajczyk (1987) for unbalanced panels. Each measure was normalized by its cross-sectional mean and standard deviation at time $t-1$. The total sample included 4975 stocks from the NYSE spanning July 1962 to December 2011.

| Measure | 1 Factor | 2 Factors | 3 Factors |
|---:|---|---|---|
| Amihud | 0.4267 | 0.5071 | 0.5912 |
| RS | 0.3819 | 0.5379 | 0.5917 |
| Roll | 0.1614 | 0.3303 | 0.3583 |
| Turnover | 0.3523 | 0.4624 | 0.4956 |
| RV | 0.2918 | 0.4289 | 0.4902 |
| GK | 0.3207 | 0.4714 | 0.5269 |
| Garch | 0.4857 | 0.6291 | 0.6993 |
| Returns | 0.2423 | 0.2621 | 0.2924 |

Table 2.2: AR(2) Results.
AR(2) estimates for the first factor of each liquidity and volatility measure with the corresponding t-stat in parentheses. The common liquidity, volatility, and uncertainty factors are also estimated. The impulse response measure the fraction of a time $t$ shock that remains after 12 periods (one year). The liquidity factors exhibit higher persistence than the volatility factors.

| Measure | $\phi_1$ | $\phi_2$ | Impulse Response | Measure | $\phi_1$ | $\phi_2$ | Impulse Response |
|---|---|---|---|---|---|---|---|
| Amihud | 0.5641 | 0.4115 | 0.5695 | RV | 0.6181 | 0.1597 | 0.0685 |
| | (31.20) | (24.64) | | | (65.51) | (8.95) | |
| RS | 0.7700 | 0.1470 | 0.3504 | GK | 0.7556 | 0.1555 | 0.0850 |
| | (41.23) | (7.10) | | | (131.38) | (20.49) | |
| Roll | 0.5732 | 0.4060 | 0.5901 | Garch | 0.6383 | 0.3292 | 0.3270 |
| | (25.45) | (18.12) | | | (51.57) | (24.03) | |
| Turnover | 0.5140 | 0.4718 | 0.6011 | VOL | 0.6643 | 0.1340 | 0.2580 |
| | (28.52) | (24.99) | | | (78.39) | (10.18) | |
| LIQ | 0.1949 | -0.0541 | 0.6112 | Returns | 0.6650 | 0.1334 | 0.0000 |
| | (5.23) | (-1.54) | | | (77.96) | (10.10) | |
| Common | 0.6794 | 0.2100 | 0.5531 | | | | |
| | (68.30) | (16.48) | | | | | |

Table 2.3: Pairwise Contemporaneous Canonical Correlations (Raw Series). Three common factors are extracted from a variety of liquidity and volatility measures. The liquidity measures are the Amihud (2002) measure, the relative spread, the Roll (1984) measure, and the turnover. Monthly volatility estimates include monthly realized variance (RV), the monthly Garman-Klass estimate, and the estimates from a GARCH(1,1) model. The sample includes 4975 NYSE stocks from July 1962 to December 2011. The contemporaneous, pairwise canonical correlation for the first three raw factors of each measure is presented below.

| | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL |
|---|---|---|---|---|---|---|---|---|---|---|
| Amihud | 0.098 | | | | | | | | | |
| RS | 0.213 | 0.802 | | | | | | | | |
| Roll | 0.199 | 0.876 | 0.952 | | | | | | | |
| Turnover | 0.292 | 0.693 | 0.881 | 0.727 | | | | | | |
| RV | 0.291 | 0.741 | 0.931 | 0.891 | 0.685 | | | | | |
| GK | 0.294 | 0.745 | 0.962 | 0.861 | 0.795 | 0.972 | | | | |
| Garch | 0.218 | 0.868 | 0.867 | 0.791 | 0.777 | 0.748 | 0.773 | | | |
| LIQ | 0.139 | 0.973 | 0.929 | 0.936 | 0.985 | 0.852 | 0.865 | 0.862 | | |
| VOL | 0.408 | 0.663 | 0.955 | 0.894 | 0.765 | 0.990 | 0.995 | 0.990 | 0.846 | |
| Common | 0.230 | 0.881 | 0.977 | 0.921 | 0.958 | 0.966 | 0.963 | 0.902 | 0.984 | 0.983 |

Table 2.4: Pairwise Contemporaneous Canonical Correlations (Prewhitened using an AR(2)). Three common factors are extracted from a variety of liquidity and volatility measures. The liquidity measures are the Amihud (2002) measure, the relative spread, the Roll (1984) measure, and the turnover. Monthly volatility estimates include monthly realized variance (RV), the monthly Garman-Klass estimate, and the estimates from a GARCH(1,1) model. The sample includes 4975 NYSE stocks from July 1962 to December 2011. The contemporaneous, pairwise canonical correlation for the three pre-whitened factors of each measure is presented below.

| | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL |
|---|---|---|---|---|---|---|---|---|---|---|
| Amihud | 0.149 | | | | | | | | | |
| RS | 0.365 | 0.121 | | | | | | | | |
| Roll | 0.322 | 0.257 | 0.800 | | | | | | | |
| Turnover | 0.349 | 0.284 | 0.684 | 0.464 | | | | | | |
| RV | 0.294 | 0.111 | 0.875 | 0.756 | 0.544 | | | | | |
| GK | 0.305 | 0.093 | 0.890 | 0.727 | 0.599 | 0.946 | | | | |
| Garch | 0.130 | 0.918 | 0.103 | 0.120 | 0.168 | 0.234 | 0.110 | | | |
| LIQ | 0.261 | 0.918 | 0.910 | 0.719 | 0.959 | 0.774 | 0.800 | 0.085 | | |
| VOL | 0.311 | 0.060 | 0.885 | 0.729 | 0.607 | 0.973 | 0.983 | 0.939 | 0.805 | |
| Common | 0.322 | 0.511 | 0.925 | 0.732 | 0.875 | 0.952 | 0.959 | 0.264 | 0.952 | 0.986 |

Table 2.5: Lead-lag Canonical Correlations Raw Factors.
Three common factors are extracted separately for multiple liquidity and volatility measures in addition to returns. Across-measure liquidity (volatility) factors were estimated for the combined liquidity (volatility) measures. Common "uncertainty" factors were extracted across all liquidity and volatility measures. The measures were standardized by their cross-sectional means and standard deviations before the factor analysis to eliminate differing units of measure. The liquidity factors considered are the Amihud (2002) measure (daily absolute return divided by the dollar volume), the relative spread (bid-ask spread divided by its mean), the Roll (1984) measure (based on the monthly autocorrelation of daily returns), and the turnover (ratio of monthly volume and shares outstanding). The volatility measures are monthly realized variance (sum of daily squared returns), the Garman-Klass measure (based on the daily high, low, open, and close), and the estimates of conditional variance of a GARCH(1,1) model. The sample includes 4975 MYSE firms over the period of July 1962 to December 2011.

| $t-1\backslash t$ | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL | Common |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Return | 0.22 | 0.20 | 0.27 | 0.26 | 0.32 | 0.29 | 0.28 | 0.34 | 0.22 | 0.47 | 0.25 |
| Amihud | 0.09 | 0.95 | 0.81 | 0.87 | 0.69 | 0.73 | 0.74 | 0.87 | 0.91 | 0.66 | 0.84 |
| RS | 0.14 | 0.80 | 0.97 | 0.86 | 0.87 | 0.85 | 0.89 | 0.90 | 0.91 | 0.88 | 0.94 |
| Roll | 0.13 | 0.87 | 0.87 | 0.92 | 0.69 | 0.81 | 0.79 | 0.82 | 0.90 | 0.80 | 0.87 |
| Turnover | 0.15 | 0.69 | 0.87 | 0.72 | 0.97 | 0.69 | 0.77 | 0.78 | 0.94 | 0.76 | 0.93 |
| RV | 0.15 | 0.71 | 0.85 | 0.81 | 0.64 | 0.87 | 0.78 | 0.83 | 0.80 | 0.86 | 0.86 |
| GK | 0.14 | 0.77 | 0.90 | 0.82 | 0.77 | 0.89 | 0.88 | 0.81 | 0.85 | 0.87 | 0.85 |
| Garch | 0.20 | 0.86 | 0.84 | 0.77 | 0.78 | 0.71 | 0.75 | 0.98 | 0.85 | 0.78 | 0.88 |
| LIQ | 0.09 | 0.91 | 0.91 | 0.89 | 0.94 | 0.82 | 0.84 | 0.88 | 0.96 | 0.83 | 0.96 |
| VOL | 0.24 | 0.67 | 0.89 | 0.80 | 0.72 | 0.88 | 0.82 | 0.87 | 0.82 | 0.91 | 0.90 |
| Common | 0.12 | 0.84 | 0.94 | 0.87 | 0.92 | 0.85 | 0.85 | 0.92 | 0.96 | 0.90 | 0.97 |

Table 2.6: Lead-lag Canonical Correlations Pre-whitened Factors.
Three common factors are extracted separately for multiple liquidity and volatility measures in addition to returns. Across-measure liquidity (volatility) factors were estimated for the combined liquidity (volatility) measures. Common "uncertainty" factors were extracted across all liquidity and volatility measures. The measures were standardized by their cross-sectional means and standard deviations before the factor analysis to eliminate differing units of measure. The liquidity factors considered are the Amihud (2002) measure (daily absolute return divided by the dollar volume), the relative spread (bid-ask spread divided by its mean), the Roll (1984) measure (based on the monthly autocorrelation of daily returns), and the turnover (ratio of monthly volume and shares outstanding). The volatility measures are monthly realized variance (sum of daily squared returns), the Garman-Klass measure (based on the daily high, low, open, and close), and the estimates of conditional variance of a GARCH(1,1) model. The sample includes 4975 MYSE firms over the period of July 1962 to December 2011.

| $t-1\backslash t$ | Return | Amihud | RS | Roll | Turnover | RV | GK | Garch | LIQ | VOL | Common |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Return | 0.16 | 0.39 | 0.23 | 0.32 | 0.20 | 0.22 | 0.25 | 0.34 | 0.40 | 0.32 | 0.24 |
| Amihud | 0.20 | 0.21 | 0.13 | 0.14 | 0.22 | 0.13 | 0.08 | 0.23 | 0.30 | 0.06 | 0.28 |
| RS | 0.14 | 0.20 | 0.16 | 0.12 | 0.24 | 0.26 | 0.19 | 0.36 | 0.19 | 0.20 | 0.24 |
| Roll | 0.18 | 0.23 | 0.10 | 0.13 | 0.19 | 0.23 | 0.11 | 0.31 | 0.14 | 0.18 | 0.21 |
| Turnover | 0.15 | 0.15 | 0.15 | 0.15 | 0.25 | 0.22 | 0.18 | 0.29 | 0.23 | 0.18 | 0.20 |
| RV | 0.11 | 0.17 | 0.17 | 0.16 | 0.27 | 0.41 | 0.20 | 0.48 | 0.15 | 0.26 | 0.22 |
| GK | 0.16 | 0.27 | 0.19 | 0.21 | 0.26 | 0.39 | 0.29 | 0.34 | 0.24 | 0.29 | 0.31 |
| Garch | 0.13 | 0.15 | 0.16 | 0.16 | 0.13 | 0.17 | 0.11 | 0.22 | 0.16 | 0.21 | 0.21 |
| LIQ | 0.19 | 0.26 | 0.15 | 0.18 | 0.24 | 0.21 | 0.17 | 0.37 | 0.32 | 0.16 | 0.31 |
| VOL | 0.18 | 0.16 | 0.21 | 0.20 | 0.25 | 0.36 | 0.19 | 0.39 | 0.18 | 0.26 | 0.29 |
| Common | 0.18 | 0.26 | 0.19 | 0.16 | 0.25 | 0.33 | 0.22 | 0.41 | 0.26 | 0.26 | 0.27 |

Table 2.7: Percent of firms with significant exposure to the common uncertainty factor.
Each measure is regressed on the common uncertainty factor, the across-measure liquidity or volatility factor, and its own measure-specific factor. Each factor is pre-whitened using an AR(2) specification. The measure-specific and liquidity/volatility factors were orthogonalized to the common uncertainty factor using a specification similar to Equation (2.10).The table reports the percentage of firms with a significant coefficient at the 5% level. It also includes the percentage of firms where the test of joint significance exceeds the 5% level. The average $R^2$ is also included. The sample contains 4975 NYSE firms over the period from July 1962 to December 2011.

| Variable | LIQ/VOL measure | Common measure | Specific measure | Joint Sign. | Average $R^2$ |
|---|---|---|---|---|---|
| Amihud | 3.0 | 5.2 | 20.1 | 14.5 | 0.05 |
| RS | 9.8 | 42.3 | 42.7 | 61.2 | 0.11 |
| Roll | 11.5 | 17.0 | 9.3 | 26.5 | 0.08 |
| Turnover | 8.3 | 33.4 | 22.7 | 35.4 | 0.06 |
| RV | 16.7 | 56.4 | 46.6 | 73.5 | 0.22 |
| GK | 23.2 | 59.6 | 18.1 | 63.4 | 0.19 |
| GARCH | 51.2 | 22.1 | 23.4 | 49.9 | 0.08 |

Table 2.8: Portfolios formed by sorting on the common uncertainty factor. Across-measure common factors (which we refer to as uncertainty factors) are jointly extracted for various liquidity and volatility measures. Each stock is then assigned to one of 30 portfolios based on its exposure to this common factor over the previous 36 months (a minimum of 24 observations is required). The excess returns for these 20 portfolios are then regressed on the FF4 factors (MKT, HML, SMB, and UMD) and the liquidity, volatility, and uncertainty factors. The liquidity measures considered are the Amihud (2002) measure, defined as the absolute return divided by dollar volume, the relative spread, the Roll (1984) measure, and turnover. The volatility measures are realized variance, the Garman and Klass (1980) estimate, and the conditional volatility estimated from a simple GARCH(1,1) model. Before extracting a common "uncertainty" factor across all of these measures, they are each standardized by their respective cross-sectional means and standard deviations. The sample consists of 4975 NYSE stocks from July 1962 to December 2011.

| Portfolio Ranking | Excess Return | t-stat | FF4 $\alpha$ | t-stat | $\beta_{\text{LIQ}}$ | t-stat | $\beta_{\text{VOL}}$ | t-stat | $\beta_{\text{ALL}}$ | t-stat |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.0195 | 4.5046 | 0.0194 | 3.9793 | 0.3005 | 2.5683 | 0.1197 | 1.6427 | 0.0407 | 1.3721 |
| 2 | 0.0147 | 4.4043 | 0.0144 | 3.8573 | 0.0607 | 0.6934 | -0.0186 | -0.4086 | 0.0187 | 0.8644 |
| 3 | 0.0118 | 4.1024 | 0.0103 | 3.3424 | -0.0120 | -0.1517 | -0.0452 | -1.0335 | 0.0085 | 0.5515 |
| 4 | 0.0111 | 4.2796 | 0.0099 | 3.6586 | -0.0289 | -0.4320 | -0.0322 | -0.7852 | 0.0118 | 1.0319 |
| 5 | 0.0064 | 2.5426 | 0.0056 | 2.0438 | -0.0515 | -0.8627 | -0.0471 | -1.3493 | 0.0149 | 1.0463 |
| 6 | 0.0097 | 3.8909 | 0.0085 | 2.9485 | -0.0571 | -0.8448 | -0.0558 | -1.3951 | 0.0117 | 0.8073 |
| 7 | 0.0089 | 3.6661 | 0.0079 | 2.9390 | -0.0582 | -1.0794 | -0.0403 | -1.2108 | 0.0105 | 0.9081 |
| 8 | 0.0065 | 2.9962 | 0.0060 | 2.3881 | -0.0937 | -1.8569 | -0.0555 | -1.7720 | 0.0121 | 0.9311 |
| 9 | 0.0089 | 4.1059 | 0.0077 | 3.0825 | -0.1023 | -1.8623 | -0.0613 | -1.8109 | 0.0110 | 1.0282 |
| 10 | 0.0080 | 3.7476 | 0.0079 | 3.2045 | -0.0951 | -1.7182 | -0.0584 | -1.7789 | 0.0101 | 0.9057 |
| 11 | 0.0079 | 3.8086 | 0.0071 | 3.0728 | -0.0715 | -1.3453 | -0.0465 | -1.4105 | 0.0115 | 1.1550 |
| 12 | 0.0080 | 3.8019 | 0.0074 | 3.1624 | -0.1392 | -2.8585 | -0.0590 | -2.0424 | 0.0127 | 1.3342 |
| 13 | 0.0075 | 3.6358 | 0.0074 | 3.2140 | -0.1157 | -2.0838 | -0.0655 | -2.0024 | 0.0133 | 1.5208 |
| 14 | 0.0073 | 3.5868 | 0.0067 | 3.1835 | -0.0913 | -2.1041 | -0.0376 | -1.4407 | 0.0105 | 1.3345 |
| 15 | 0.0078 | 3.8780 | 0.0070 | 3.2427 | -0.1171 | -2.4847 | -0.0551 | -1.8563 | 0.0075 | 0.9405 |
| 16 | 0.0068 | 3.4204 | 0.0060 | 2.7278 | -0.0943 | -2.1498 | -0.0385 | -1.5320 | 0.0170 | 2.0419 |
| 17 | 0.0098 | 4.7877 | 0.0094 | 4.1378 | -0.1477 | -3.2179 | -0.0572 | -2.1802 | 0.0049 | 0.5834 |
| 18 | 0.0084 | 4.0529 | 0.0074 | 3.2429 | -0.1019 | -2.2914 | -0.0418 | -1.5501 | 0.0105 | 1.1511 |
| 19 | 0.0085 | 4.1794 | 0.0083 | 3.8859 | -0.1434 | -3.3938 | -0.0580 | -2.3862 | 0.0166 | 2.3591 |
| 20 | 0.0091 | 4.5246 | 0.0081 | 3.8605 | -0.1403 | -3.4548 | -0.0458 | -1.8923 | 0.0096 | 1.2387 |
| 21 | 0.0089 | 4.2084 | 0.0083 | 3.4783 | -0.1248 | -3.1453 | -0.0439 | -1.7829 | 0.0112 | 1.2800 |
| 22 | 0.0086 | 4.1657 | 0.0078 | 3.3650 | -0.1514 | -3.7293 | -0.0553 | -2.5299 | 0.0102 | 1.1763 |
| 23 | 0.0081 | 3.7887 | 0.0073 | 3.1654 | -0.1604 | -3.9728 | -0.0515 | -2.1747 | 0.0091 | 1.0759 |
| 24 | 0.0098 | 4.5008 | 0.0087 | 3.7648 | -0.1666 | -3.2526 | -0.0574 | -1.9052 | 0.0117 | 1.3508 |
| 25 | 0.0117 | 4.9659 | 0.0106 | 4.1432 | -0.1664 | -3.1707 | -0.0543 | -1.7988 | 0.0053 | 0.5836 |
| 26 | 0.0108 | 4.4781 | 0.0088 | 3.1468 | -0.2020 | -3.3677 | -0.0786 | -2.5314 | 0.0090 | 0.7700 |
| 27 | 0.0119 | 4.6788 | 0.0105 | 3.7298 | -0.1591 | -3.0534 | -0.0427 | -1.5182 | 0.0162 | 1.4848 |
| 28 | 0.0137 | 4.9547 | 0.0126 | 4.1850 | -0.1903 | -2.8202 | -0.0494 | -1.3322 | 0.0123 | 1.0475 |
| 29 | 0.0166 | 5.1151 | 0.0147 | 4.1660 | -0.2577 | -3.1010 | -0.0778 | -1.6780 | 0.0023 | 0.1551 |
| 30 | 0.0197 | 4.7515 | 0.0174 | 4.1643 | -0.2243 | -2.0289 | -0.0097 | -0.1484 | -0.0094 | -0.5549 |

Table 2.9: Pricing uncertainty in the cross-section.
Factors are extracted for multiple liquidity and volatility measures. Additionally, common liquidity (volatility) factors across all liquidity (volatility) measures are extracted. Finally, uncertainty factors are obtained from the collection of liquidity and volatility measures. Each firm is sorted into one of $m$ portfolios based on its individual exposure to the common uncertainty factor (estimated using a rolling 36 month window where firms are required to have a minimum of 24 months of observations). The results of cross-sectional regressions of excess returns on the factor loadings (betas) is presented below. Since the loadings at the firm level are nosier than the loadings at the portfolio level, each firm is assigned the vector of betas of its portfolio in month $t$. Before performing the factor analysis, each measure is standardized by its cross-sectional means standard deviations. The liquidity measures include the Amihud (2002) measure (sum of absolute returns divided by dollar volume), the relative spread (the ratio of bid-ask spread and the average of the bid and ask), the Roll (1984) measure (based on the monthly autocorrelation of daily returns), and the turnover (ratio of volume and shares outstanding). The volatility measures are realized variance (sum of squared daily returns), the Garman and Klass (1980) measure (based on the high, low, open, and close price), and the conditional volatility estimates of a monthly GARCH(1,1) model. The sample consists of 4975 MYSE stocks from July 1962 to December 2011.

| | MKT | HML | SMB | MOM | LIQ | VOL | Common |
|---|---|---|---|---|---|---|---|
| $m = 70$ | -0.4275 | | | | -0.0041 | 0.0212 | -0.0392 |
| | [-1.13] | | | | [-0.80] | [1.51] | [-1.63] |
| | -0.2337 | 0.2702 | -0.0037 | 0.5179 | -0.0033 | 0.0191 | -0.0518 |
| | [-0.56] | [0.99] | [-0.02] | [1.39] | [-0.66] | [1.59] | [-2.00] |
| $m = 50$ | -0.6805 | | | | -0.0032 | 0.0123 | -0.0363 |
| | [-1.24] | | | | [-0.65] | [0.94] | [-1.12] |
| | -0.2469 | 0.1129 | 0.1719 | 0.8210 | 0.0005 | 0.0089 | -0.0747 |
| | [-0.51] | [0.43] | [0.72] | [1.86] | [0.10] | [0.68] | [-1.80] |
| $m = 25$ | -1.4810 | | | | -0.0036 | 0.0046 | -0.0588 |
| | [-1.92] | | | | [-0.65] | [ 0.25] | [-1.38] |
| | 0.1478 | 0.8664 | -0.0908 | 1.3348 | -0.0002 | 0.0126 | -0.1486 |
| | [0.21] | [ 2.56] | [-0.32] | [1.74] | [-0.03] | [0.71] | [-2.28] |
| $m = 20$ | -1.3987 | | | | 0.0004 | -0.0079 | -0.0748 |
| | [-1.68] | | | | [0.07] | [-0.38] | [-1.72] |
| | -0.4349 | 0.3862 | 0.2551 | 0.5268 | 0.0020 | -0.0007 | -0.0779 |
| | [-0.61] | [1.02] | [0.64] | [0.89] | [0.33] | [-0.03] | [-1.57] |
| $m = 15$ | -1.6116 | | | | -0.0032 | 0.0049 | -0.0489 |
| | [-2.09] | | | | [-0.44] | [0.18] | [-1.48] |
| | -0.2018 | 0.0770 | 0.0822 | 1.2788 | 0.0033 | 0.0047 | -0.0904 |
| | [-0.23] | [0.15] | [0.23] | [1.40] | [0.43] | [0.14] | [-1.94] |

Table 2.10: Pricing uncertainty in the cross-section.
Common liquidity (volatility) factors are extracted across several liquidity (volatility) measures. Each firm is sorted into one of $m$ portfolios based on its individual exposure to either the liquidity or volatility factor (estimated using a rolling 36 month window where firms are required to have a minimum of 24 months of observations). The results of cross-sectional regressions of excess returns on the factor loadings (betas) is presented below. Since the loadings at the firm level are nosier than the loadings at the portfolio level, each firm is assigned the vector of betas of its portfolio in month $t$. Before performing the factor analysis, each measure is standardized by its cross-sectional means standard deviations. The liquidity measures include the Amihud (2002) measure (sum of absolute returns divided by dollar volume), the relative spread (the ratio of bid-ask spread and the average of the bid and ask), the Roll (1984) measure (based on the monthly autocorrelation of daily returns), and the turnover (ratio of volume and shares outstanding). The volatility measures are realized variance (sum of squared daily returns), the Garman and Klass (1980) measure (based on the high, low, open, and close price), and the conditional volatility estimates of a monthly GARCH(1,1) model. The sample consists of 4975 MYSE stocks from July 1962 to December 2011.

| # of Portfolios | MKT | HML | SMB | MOM | LIQ | VOL | # of Portfolios | MKT | HML | SMB | MOM | LIQ | VOL |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m = 60$ | -1.0234 | | | | -0.0236 | | $m = 25$ | -1.7877 | | | | -0.0234 | |
| | (-1.48) | | | | (-1.77) | | | (-1.52) | | | | (-1.37) | |
| | -0.6380 | 0.2003 | 0.3328 | 0.7853 | -0.0241 | | | -0.8625 | 0.1782 | 0.0273 | 1.1506 | -0.0305 | |
| | (-0.95) | (0.69) | (1.27) | (1.75) | (-1.73) | | | (-0.93) | (0.45) | (0.09) | (2.04) | (-1.71) | |
| | -0.7856 | | | | | -0.0378 | | -1.1995 | | | | | -0.0126 |
| | (-1.34) | | | | | (-0.67) | | (-1.52) | | | | | (-0.11) |
| | -0.8217 | -0.2311 | 0.1843 | 0.1443 | | -0.0349 | | -1.2034 | -0.0999 | 0.4174 | -0.2495 | | -0.0541 |
| | (-1.54) | (-0.99) | (0.80) | (0.38) | | (-0.60) | | (-1.72) | (-0.29) | (1.44) | (-0.48) | | (-0.50) |
| $m = 50$ | -1.1239 | | | | -0.0174 | | $m = 15$ | -2.3972 | | | | -0.0340 | |
| | (-1.54) | | | | (-1.22) | | | (-1.84) | | | | (-1.66) | |
| | -0.4606 | 0.4856 | 0.2462 | 1.0395 | -0.0284 | | | -2.2222 | -0.1255 | -0.5720 | 0.4864 | -0.0499 | |
| | (-0.71) | (1.35) | (0.93) | (1.82) | (-1.71) | | | (-2.24) | (-0.23) | (-1.02) | (0.63) | (-2.10) | |
| $m = 45$ | -1.1194 | | | | | -0.0806 | | -1.1460 | | | | | -0.0684 |
| | (-1.65) | | | | | (-1.24) | | (-1.00) | | | | | (-0.62) |
| | -1.0810 | -0.4134 | 0.2184 | 0.3855 | | -0.1073 | | -0.3014 | 0.0449 | 0.1269 | 0.9159 | | -0.1737 |
| | (-1.71) | (-1.45) | (0.75) | (1.05) | | (-1.57) | | (-0.34) | (0.06) | (0.28) | (1.40) | | (-1.53) |

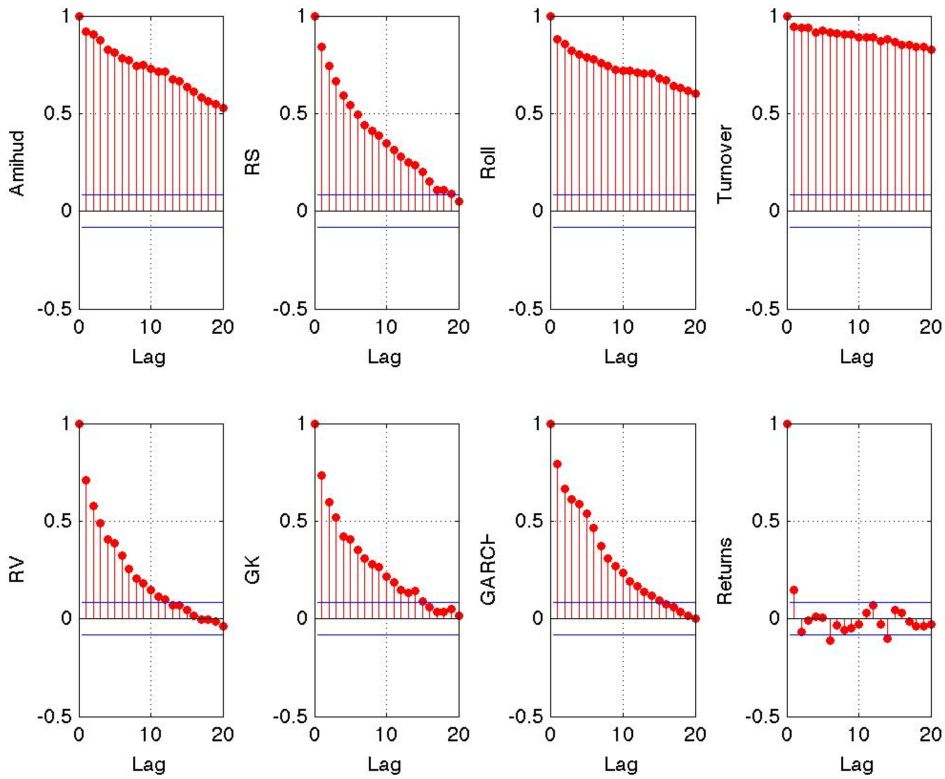



FIGURE 2.1: Autocorrelations of liquidity and volatility factors.
Common factors are extracted for liquidity measures, volatility measures, and returns. The autocorrelations of the first factor for each measure are depicted in the below plots. The liquidity measures are the Amihud measure, relative spread (RS), the Roll measure, and turnover. The measures of volatility include monthly realized variance (RV), the Garman-Klass (GK) estimate, and monthly GARCH(1,1) estimates. The sample includes 4975 NYSE stocks from July 1962 to December 2011.

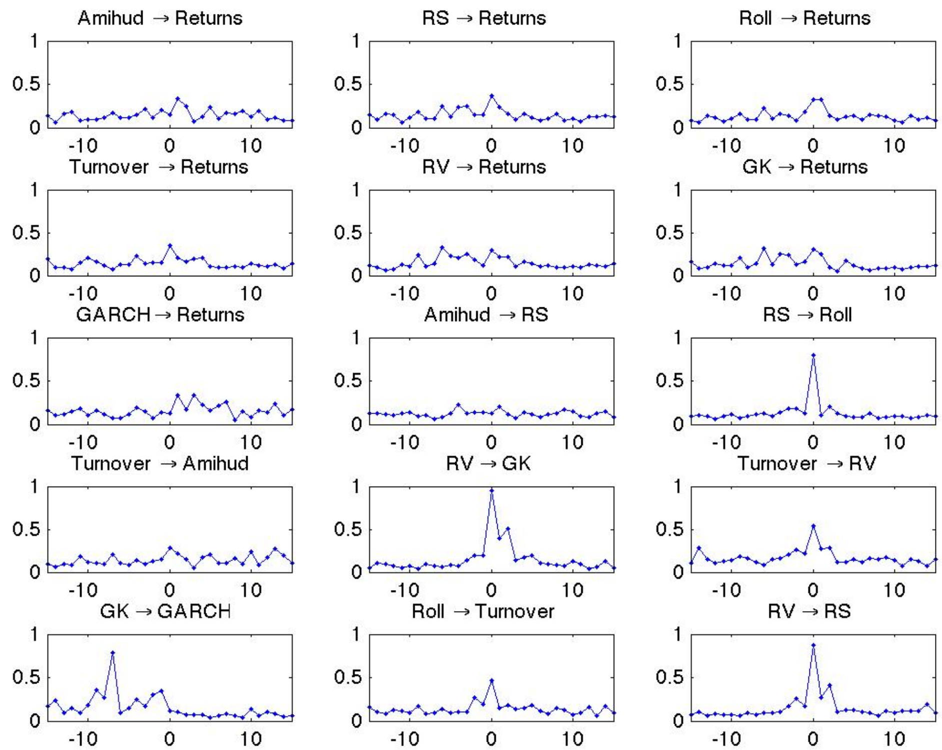



FIGURE 2.2: Canonical lead-lag correlations using the first three factors. Common factors are extracted for liquidity measures, volatility measures, and returns. Pairwise lead-lag canonical correlations (for 15 leads and lags) of the first 3 factors for each measure are plotted below. Each measure is standardized by its mean and standard deviation before performing the factor analysis. Additionally, the factors are pre-whitened using an AR(2) specification. The liquidity measures are the Amihud measure, relative spread (RS), the Roll measure, and turnover. The measures of volatility include monthly realized variance (RV), the Garman-Klass (GK) estimate, and monthly GARCH(1,1) estimates. The sample includes 4975 NYSE stocks from July 1962 to December 2011.

# 3

# Can Stock Market Liquidity and Volatility Predict Business Cycles?

## 3.1 Introduction

Recently there has been a large branch of literature that examines market liquidity. Pástor and Stambaugh (2003), Acharya and Pedersen (2005), Chen (2005), and Sadka (2006) all look into systematic liquidity risk. Additionally, since there are several different measures of liquidity, many studies have focused on identifying a common systematic liquidity factor (see Chordia et al. (2000), Hasbrouck and Seppi (2001), Eckbo and Norli (2002), and Korajczyk and Sadka (2008)). With the recent financial crisis, there has been an interest in the apparent causal link between a reduction in liquidity and an economic slowdown.

In a paper by Næs et al. (2011), they show that this link between liquidity and recessions is not a recent phenomenon but has existed in past recessions as well. They find that on average there is an increase in illiquidity prior to a recession followed by an increase in liquidity during the tail end of a recession. Furthermore, measures of liquidity help in forecasting future real GDP growth. In addition to linking liquidity to business cycles, several papers have also explored stock market volatility and its relation to real macroeconomic variables and business cycles (see Schwert (1989), Schwert (1990), and Hamilton and Lin (1996)). Hamilton and Lin (1996) find that stock volatility may be useful in forecasting economic activity.

I merge these two trains of thought and see if there is any added benefit from considering liquidity and volatility jointly. In chapter 2, multiple daily liquidity and volatility measures are estimated from daily stock prices and returns. Common, what I will term as "uncertainty," factors are extracted across all of the liquidity and volatility measures. I find that this common risk factor carries a significant premium and helps explain the cross section of expected returns. In this chapter, I explore a possible link to this uncertainty measure and the real economy and business cycles.

When plotting real GDP growth and shocks to our uncertainty measure before, during, and after recessions, I find that, on average, in quarters preceding a recession there are positive shocks to uncertainty. Similarly, on average at the beginning of a recession there are positive shocks to uncertainty. However, towards the end of a recession and in the quarters following a recession there are, on average, negative shocks to our uncertainty measure. In-sample test results indicate that this uncertainty measure helps predict several macroeconomic variables including real GDP growth, growth in industrial production, CPI growth, real consumption growth and changes in real investment. Additionally, out-of-sample forecasting tests indicate that a forecasting model including the uncertainty measure outperforms an AR(1) forecasting model for real GDP growth. When comparing the out-of-sample performance between models with our uncertainty measure and models involving just liquidity measures, I find there is no statistical difference between their expected mean squared forecast error (MSFE). Our results suggest that while there is a definite link between our uncertainty measure and the real economy, it doesn't appear to offer an improvement over liquidity measures in forecasting business cycles.

The paper is organized as follows. Section 2 discusses the specific liquidity and volatility measures as well as the method for extracting the risk factor, Section 3 presents the in-sample and out-of-sample forecasting tests, and Section 4 concludes.

## 3.2 Data

### *3.2.1 Liquidity and Volatility Measures*

Similar to chapter 2, this chapter utilizes data from the daily CRSP databases for stocks traded on the NYSE. The time range is from January 1947 to December 2012. Since trading on the NASDAQ uses a different trading mechanism relying heavily on market makers, only stocks traded on the NYSE are considered in the analysis. Additionally, only assets with a CRSP share code of 10 or 11 (ordinary common shares) are considered, which will eliminate certificates, Americus Trusts components, ADRs, shares of beneficial interest, closed-end funds, REIT's, and ETFs. Stocks with a price lower than \$1 are excluded as well as those observations with a volume = 0. After appropriate filtering, we are left with a total of 5281 firms over a total of 264 quarters.

There are a wide range of proposed measures of liquidity. We implement a total of four liquidity measures at a quarterly frequency from the daily stock data. The first is the measure based on Amihud (2002). Define the Amihud measure for stock $i$ in month $t$ as

$$A_{i,t} = \frac{1}{d_t} \sum_{j=1}^{d_t} \frac{|r_{i,j}|}{dvol_{i,j}} \tag{3.1}$$

where $r_{i,j}$ is the return on asset $i$ on day $j$ of quarter $t$, $d_t$ is the number of trading days in the quarter, and $dvol_{i,j}$ is the dollar volume for asset $i$ on day $j$ of quarter $t$. Following both Acharya and Pedersen (2005) and Korajczyk and Sadka (2008), the quarterly measure $A_{i,t}$ is scaled by the ratio of the market capitalization of the CRSP market index at time $t-1$ and at the reference date of July 1962. In order for the quarterly measure to be included in the sample, a stock is required to have at least 45 daily observations in the quarter. This measures the price impact of trades. Suppose you see a large price change (high numerator) for a low volume trade (a small denominator). This would represent an illiquid asset and would correspond with a large value of the Amihud measure.

The second liquidity measure employed is the turnover, the ratio of quarterly volume and shares outstanding. It is defined as

$$TO_{i,t} = \frac{\sum_{j=1}^{d_t} vol_{i,j}}{SO_{i,t}} \tag{3.2}$$

where $SO_{i,t}$ is the number of shares outstanding at the end of quarter $t$. Once again, it is required that a stock have at least 45 daily observations in quarter $t$ to be included in the sample.

The relative spread is calculated as the difference between the bid and the ask divided by the midpoint price (average of the bid and ask).

$$RS_{i,t} = \frac{1}{d_t} \sum_{j=1}^{d_t} \frac{Ask_{i,j} - Bid_{i,j}}{midpt_{i,j}} \tag{3.3}$$

This is calculated at the daily frequency and then aggregated by taking the quarterly average of the daily measures. The purpose of the relative spread is to measure the implicit cost of trading a small number of shares.

The final liquidity measure employed is that of Roll (1984). Assuming the existence of a constant spread $s$, Roll shows that the spread can be estimated as $\hat{s} = 2\sqrt{-Scov}$ where $Scov$ is the covariance of adjacent daily returns. This is estimated each quarter using daily returns where a minimum of 45 daily returns is required to be included. Since this is undefined when $Scov > 0$, whenever a stock has $Scov > 0$ for a given quarter the Roll measure is set to missing for that quarter as in Næs et al. (2011)[1].

Two different estimates of quarterly volatility are employed in the following analysis. The first is an estimate formed from the daily realized variance measure simply defined as

$$RV_{i,t} = \sum_{j=1}^{d_t} r_{i,j}^2 \tag{3.4}$$

where, again, $r_{i,j}$ is the return of asset $i$ on day $j$ of quarter $t$ and $d_t$ is the number of trading days in quarter $t$.

The other estimate of quarterly volatility for each asset is obtained by estimating a simple GARCH(1,1) model over an expanding window with a minimum of 8 quarterly returns required for estimation. Formally, the monthly variance for our GARCH(1,1) model is defined as

$$r_t = c + \epsilon_t \qquad \epsilon \sim N(0, \sigma_t^2) \tag{3.5a}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_i \sigma_{t-1}^2 \tag{3.5b}$$

In order to reduce the effects of outliers, each quarterly estimate of liquidity and volatility is Windsorized at the 1st and 99th cross-sectional percentiles for each quarter[2].

This results in an unbalanced panel of 4 liquidity and 2 volatility measures over 5281 NYSE firms spanning a total of 264 quarters. The various liquidity (volatility) measures will be used to derive a common liquidity (volatility) factor. A common cross-sectional factor will be extracted from the combined liquidity and volatility measures which we will refer to as the common uncertainty factor.

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[1] Harris (1990) suggests using $\hat{s} = -2\sqrt{Scov}$ when $Scov < 0$, but this would result in a negative spread which would imply a negative transaction cost.

[2] To illustrate, consider the variance estimate $RV_{i,t}$. Let $RV_t^{99\%}$ be the 99th percentile of all RV estimates for the quarter $t$. If $RV_{j,t} > RV_t^{99\%}$ then $RV_{j,t}$ is set equal to $RV_t^{99\%}$. Similarly, any quarterly measure that is less than $RV_t^{1\%}$ will be set equal to the 1st percentile.

### 3.2.2 Macroeconomic Data

The primary series that is explored in this paper is real GDP growth calculated as the log difference of the quarterly real GDP[3]. Other macroeconomic variables considered in this paper are industrial production (IP), CPI, unemployment rate (UE), real personal consumption expenditure (Cons), and real gross private domestic investment (Inv). Additionally, we also use the Term spread and Credit spread (Cred). Term is calculated as the difference between the yield on the 10-year Treasury bond and the yield on the 3-month Treasury bill, and Cred is the difference between the yield on Moody's Baa rated bonds and the yield on a 30-year government bond. All of this data was taken from the Federal Reserve Economic Data (FRED) available through the Federal Reserve Bank of St. Louis, although the credit spread is only partially available and as such is only included in some of the analysis.

### 3.2.3 Factor decomposition of Liquidity and Volatility Measures

We will be examining the common uncertainty factor across the various liquidity and volatility measures using a process similar to that of Korajczyk and Sadka (2008). Since the units are not comparable for the various liquidity and volatility measures, each measure is standardized using the mean and standard deviation in the cross section using all available data prior to quarter $t$. Specifically, let $M^i$ be the $n \times T$ matrix of estimator $i$ (this could be either a liquidity or a volatility estimator). Define $\hat{\mu}^i_{t-1}$ and $\hat{\sigma}^i_{t-1}$ as the cross-sectional mean and standard deviation for measure $i$ estimated for all of the sample up to $t-1$. Then the standardized measure is calculated as $S^i_{j,t} = (M^i_{j,t} - \hat{\mu}^i_{t-1})/\hat{\sigma}^i_{t-1}$. The estimator $S^i$ is assumed to follow the factor model

$$S^i = B^i F^i + \epsilon^i, \tag{3.6}$$

[3] Real GDP is taken from the GDPC96 series which is the real GDP taken to 3 decimals, in billions of chained 2005 dollars, and seasonally adjusted.

where $F^i$ is a $k \times T$ matrix of shocks to the liquidity (volatility) measure that are common across the set of $n$ assets, $B^i$ is a $n \times k$ matrix of sensitivities to the common factor, and $\epsilon^i$ is the $n \times T$ matrix of asset specific shocks to the liquidity (volatility) measure. Connor and Korajczyk (1986) show that $n$-consistent estimates of the factors, $F^i$, are obtained by calculating the eigenvalues of

$$\Omega^i = \frac{S^{i\prime} S^i}{n}. \tag{3.7}$$

While this estimator relies on a balanced panel, it does vastly simplify the calculations as we are now simply calculating the eigenvectors of a $T \times T$ matrix which is independent of the number of stocks in our sample. In order to accommodate the fact that our panel is unbalanced, we follow the estimation technique of Connor and Korajczyk (1987) which will essentially estimate the elements of $\Omega$ using only the observed data. In order to implement this method, all of the missing observations in $S^i$ are replaced with zeros and the resulting balanced panel will be called $S^{i*}$. Define $N^i$ as a $n \times T$ indicator matrix where each element takes a value of 1 if the element in $S^i$ is observed or 0 if the corresponding element in $S^i$ is missing. Now we can construct an unbalanced equivalent of $\Omega$ that only uses the cross-sectional averages of the observed data.

$$\Omega^{i,u}_{t,\tau} = \frac{(S^{i*\prime} S^{i*})_{t,\tau}}{(N^{i\prime} N^i)_{t,\tau}} \tag{3.8}$$

The estimates of the $k$ latent factors, $\hat{F}^i$, can be calculated as the eigenvectors $(T \times 1)$ of the $k$ largest eigenvalues of $\Omega^{i,u}$. Following Connor and Korajczyk (1986), the eigenvectors are normalized so that the rows have a mean-square of 1.

Common factors across all of the liquidity (volatility) measures are extracted. This can be accomplished by stacking the multiple liquidity (volatility) measures and then using the stacked matrix to form $\Omega$. The factors extracted from the stacked liquidity (volatility) measures will be referred to as the common, or across-measure, liquidity (volatility) factors. The sign of the liquidity factors is chosen so that an increase in the factor will correspond to an increase in liquidity. This is done by choosing the sign so that the within-measure factors are negatively correlated with the cross-sectional mean of the measure (although for turnover it will be positively correlated). In addition to across-measure liquidity and volatility factors, a common "uncertainty" factor is extracted using all of the quarterly liquidity and volatility measures. The majority of the analysis will center on examining the relationship between this uncertainty factor and the macroeconomy, with special attention given to business cycles.

## 3.3 Results

### *3.3.1 Data Transformation*

Frequently, there is a need to transform macroeconomic series due to the presence of a unit root. We perform various Augmented Dickey-Fuller (ADF) tests to determine whether a variable contains a possible unit root, including specifications with and without trend components. Not surprisingly, we fail to reject the null that the series does not possess a unit root for the macroeconomic variables not including the Term spread. The presence of a unit root in the series of shocks to the uncertainty, liquidity, and volatility factors was also rejected by the ADF test. We are also potentially interested in the various liquidity measures upon which our factors are based. We therefore calculate the equally-weighted cross-sectional quarterly mean for both the Amihud and Roll measures.

In order to achieve stationarity in the series used in this study, we transform the necessary variables by taking the first log difference. For example, we construct dGDP (real GDP growth) as dGDP = $\ln\left[\frac{GDP_t}{GDP_{t-1}}\right]$. The other variables for which we could not reject the presence of a unit root undergo a similar log-difference transformation as in Næs et al. (2011).

### *3.3.2 Uncertainty Shocks and Business Cycles*

It is widely accepted that in the most recent recession there was a strong connection between the decline in liquidity in financial markets and the financial crisis. We also saw an increase in the volatility of returns. We begin our exploration of this relationship by first examining Figure 3.1. In this figure we follow the method of Næs et al. (2011) in constructing a bar graph of the accumulated average quarterly real GDP growth before and after the NBER defined recession. For each of the recessions in the sample, there are a total of 11 defined NBER recessions between 1947Q1 and 2012Q4, we construct a window the begins 5 quarters before the date of the peak (beginning of the recession) and extend that for 5 quarters after the recession ends. We then average the growth rates across the recessions, remembering that each is aligned so that N1 is the first quarter of the recession on the x-axis of Figure 3.1. The average growth rates are then accumulated over the even window. The average shocks to our uncertainty measure (as derived from a variety of liquidity and volatility measures) during the same window is also included in Figure 3.1.

As we see, over all of the recessions since 1947Q1, on average there is positive growth to real GDP in the 5 quarters leading up to the recession. During the recession, the average growth rate becomes negative and then begins improving in the quarters after the recession, exactly as we would expect given the definition of an economic recession. The true interest in Figure 3.1 is the behavior of the average shocks to our uncertainty measure around the NBER recession dates. We see a pattern in the uncertainty shocks where there are positive shocks (increased uncertainty) preceding the recession and in the beginning of the recession. Then, on average, the shocks become negative (decreased uncertainty) as the economy begins to leave the recession. This seems to suggest a possible relationship between our measure of uncertainty and changes in real GDP growth.

### *3.3.3 Correlations*

In Table 3.1, we present the contemporaneous correlations between the US macroeconomic variables as well as our measures of uncertainty, liquidity, and volatility. A brief examination of the table reveals some interesting relationships. Our uncertainty measure is negatively correlated with several key macroeconomic series including real GDP growth (dGDP), the growth in real industrial production (dIP), real consumption growth (dCons), and growth in real private investment (dInv). This suggests that as uncertainty increases (in other words, we see a decrease in assets' liquidity and an increase in the volatility of stock returns) the growth rates of real GDP, real consumption, real private investment, and real industrial production will decrease. Should there be a large enough spike in uncertainty, the economy may slide into a recession. Another interesting feature of Table 3.1 is that the shocks to the across-measure volatility factor (VOL) do not appear to be significantly correlated to any of the series except for the uncertainty measure. This may suggest that there is little improvement in the forecasting of GDP growth by including the volatility measures in addition to liquidity measures (i.e. forecasts from a model with UNC may not be significantly better than those from a model using LIQ). Recall that LIQ measures liquidity while both Amihud and Roll measure illiquidity which explains the opposite signs on their correlations. The signs of the correlations for both the term spread (Term) and the credit spread (dCred) are what we would expect. Furthermore, nothing unusual appears in the correlations for the macroeconomic variables.

### *3.3.4 In-sample Predictability*

In this subsection, we explore both the ability of our uncertainty measure to predict GDP growth in-sample as well as any Granger causality between the series. The models under consideration are of the form

$$y_{t+1} = \alpha + \beta UNC_t + \gamma' X_t + \eta_{t+1} \tag{3.9}$$

where $UNC_t$ is our uncertainty measure and $X_t$ is a matrix of additional regressors including $Term_t$ and the lagged dependent variable. The dependent variable we are primarily interested in is real GDP growth, but we also include the growth in the unemployment rate (dUE), the growth in real industrial production (dIP), real consumption growth (dCons), and the growth in private investment (dInv).

The results of the various regressions are presented in Table 3.2. We first consider models that include only $UNC_t$ and the lagged dependent variable. Additional regressions are performed with the inclusion of Term as an additional regressor.

One important note is that $\hat{\beta}$ is significant in every specification suggesting that shocks to our proposed uncertainty measure help predict the gap growth in the following quarter. Specifically, a positive shock (increase) in "uncertainty" (corresponding with a decrease in liquidity and an increase in volatility) predicts lower GDP growth, lower Industrial Production growth (dIP), increased growth in the unemployment rate, decreased growth in real consumption, and reduced growth in real private investment. Consider the following illustration to better understand the coefficients. Suppose we have a one standard deviation change in uncertainty measure. The standard deviation of $UNC$ is 0.1801. Thus, this 1 standard deviation increase would result in a predicted $0.1801 \times -0.010 = -0.0018$ or 0.18% drop in the quarterly real GDP growth. The average quarterly real GDP growth over our sample period is 0.78% which means a 1 standard deviation increase would lower the real GDP growth forecast by about 24% of its historical average. Similar to the contemporaneous correlations, the $\hat{\beta}$'s have the expected signs. A positive shock to our uncertainty measure will result in a lower forecast for real GDP growth (dGDP), real industrial production growth (dIP), growth in real consumption expenditures (dCons), and growth in real private investment (dInv). It will also lead to an increased forecast of the growth in the unemployment rate (dUE). While chapter 2 shows that the commonality between liquidity and volatility risk (our uncertainty measure) carries a significant risk premium for investors, we show that changes in the uncertainty measure impact predictions of future economic growth not limited to the financial sector.

### *3.3.5 Granger Causality*

In addition to looking at using our uncertainty measure to help predict various macroeconomic series, we will examine a possible causal relationship going in the opposite way as well. Næs et al. (2011) performed several Granger causality tests to better understand the relationship between GDP growth and 3 different measures of liquidity. They found evidence of a one-way Granger casualty from liquidity measures to GDP growth. We will similarly examine whether macroeconomic conditions affect our uncertainty measure or if it is primarily a one direction causal relationship.

The Granger causality tests are performed using a VAR setup where the optimal number of lags for each VAR system was chosen using the Bayesian Information Criterion (BIC)[4]. The tests were performed over the entire sample as well as the two subsamples created by splitting the sample in half. Table 3.3 contains the results of the tests for Granger causality between various macroeconomic variables and our uncertainty measure derived from liquidity and volatility measures.

Examining Table 3.3, we notice strong evidence that our uncertainty measure Granger causes real GDP growth and real Industrial Production growth but the Granger causal relationship does not run in the opposite direction. What is surprising is that the relationship doesn't hold for the second half of the sample for growth in the unemployment rate and real consumption growth even though there is evidence

[4] The BIC chose VAR systems with one lag for each test.

that $\text{UNC}_t$ Granger causes those variables for the entire sample and the first half of the sample. It may be interesting in future work to gain a better understanding of the reason for this breakdown.

### *3.3.6 Out-of-Sample Predictability*

Up to this point, our analysis has focused on the in-sample predictive power of UNC for various macroeconomic series. In this subsection, we will evaluate the out-of-sample performance of forecasts for real GDP growth that rely on our uncertainty measure (UNC). Specifically, we will compare several nested and non-nested models to determine if there is any statistically significant gains to including UNC in the forecasting models for real GDP growth.

Before discussing the statistical tests for out-of-sample forecast evaluation, we will discuss the general methodology for constructing our forecasts. The forecasts are calculated by first estimating the model over a rolling, fixed 5 year window (20 quarters). When constructing the forecast from the estimated parameters, we use financial variables (e.g. UNC) from the previous quarter but GDP growth is lagged 2 quarters due to the delay in reporting its most recent value (see Næs et al. (2011)). This means the first out-of-sample forecast is for 1952Q2 using parameters from the regression spanning 1947Q1 to 1952Q1. The financial variables for the 1952Q2 forecast are from 1952Q1 while the lagged GDP growth value is from 1951Q4. From there everything is shifted one quarter, repeated, and then shift forward again.

When comparing non-nested models, we rely on the Diebold and Mariano (1995) statistic (DM) while nested models are compared using the encompassing test proposed by Clark and McCracken (2001)[5]. The DM test statistic tests the null hypothesis of equal predictive accuracy. Let $d_t = L(\epsilon^1_{t+h|t}) - L(\epsilon^2_{t+h|t})$ where $L(\cdot)$ is

[5] The test for equal mean squared forecast error between two nested models proposed by McCracken (2007) yields similar results.

simply the squared loss function. Then the null of equal predictive accuracy can be rewritten as $H_0 : E[d_t] = 0$.

Now let us look more closely at the test for nested models. The ENC-NEW test proposed by Clark and McCracken (2001) tests whether the restricted model (the model with fewer regressors, in our case the model without UNC) encompasses the unrestricted model. If we reject the null hypothesis that the restricted model encompasses the unrestricted model, then we would conclude that the additional regressors improve the accuracy of the forecasts. The test statistic is given by

$$\text{ENC-NEW} = (P - h + 1)\frac{P^{-1}\sum_t[\hat{u}_{r,t+1}^2 - \hat{u}_{r,t+1} \cdot \hat{u}_{u,t+1}]}{MSFE_u} \tag{3.10}$$

where $P$ is the number of out-of-sample forecasts, $h$ is the forecast horizon, $\hat{u}_{r,t+1}$ denotes the out-of-sample forecast errors for the restricted model, and $MSFE_u$ is the mean squared forecast error for the unrestricted model. The ENC-NEW test statistic has a nonstandard asymptotic distribution so we use the bootstrapped critical values provided by Clark and McCracken (2001). Table 3.4 presents the results of the out-of-sample forecasting tests.

The DM tests of non-nested models allows us to test whether there is any significant out-of-sample forecasting gain by using our uncertainty measure based on liquidity and volatility measures versus a liquidity measure based upon multiple liquidity measures or the raw liquidity measures themselves. While the previous tests have shown there is a connection between our uncertainty measure and macroeconomic variables, the DM tests conclude that the expected MSFE of forecasts based on UNC, LIQ, and the quarterly mean of the Amihud measure is equal.

The nested model tests show that there is definitely a benefit to including either UNC or LIQ as an additional regressor into an AR(1) forecasting model for GDP growth (dGDP), growth in industrial production (dIP), and growth in the unemployment rate (dUE) but not for consumption growth (dCons). This indicates that our uncertainty measure is useful in forecasting several macroeconomic variables that are indicative of whether the economy is in a recession. Also, the top right number in the lower panel of Table 3.4, 1.26, indicates that there is no significant forecasting gain to adding our uncertainty measure to a model that already included the common liquidity (LIQ) factor and lagged dGDP. This further confirms that while the uncertainty measure is certainly useful in predicting changes to real GDP growth, growth in industrial production, and growth in the unemployment rate, and it displays a strong correlation with several macroeconomic variables, it doesn't appear to significantly outperform LIQ when forecasting real GDP growth.

## 3.4 Conclusion

With the recent financial crisis, during which there was a noticeable link between the economic downturn and a reduction in liquidity, there has been a lot of research focusing on measuring liquidity and linking it to the overall state of the economy. Næs et al. (2011) show that a link between liquidity and GDP growth has existed in past recessions and isn't limited to this most recent crisis. In this paper, we utilize the uncertainty measure of chapter 2 which is based on multiple measures of stock liquidity and volatility. Chapter 2 examined the impact of this uncertainty measure on the cross section of expected returns, and found that the commonality between liquidity and volatility risk is what carries the significant risk premium (as opposed to simply liquidity or volatility).

The construction of this quarterly "uncertainty" measure relies on daily measures of liquidity and volatility and is based on the work of Korajczyk and Sadka (2008) who analyze several liquidity measures. It relies on various liquidity and volatility measures across 5281 NYSE firms from January 1947 to December 2012. A latent factor model is estimated across the collection of all liquidity and volatility measures, and from these factors we obtain a measure of the commonality of liquidity and volatility which we call "uncertainty." We then explore a possible link between real economic variables and this uncertainty measure. We find that the uncertainty measure exhibits both in-sample and out-of-sample predictive ability for real GDP growth. Additionally, when examining the average shock to uncertainty with the average quarterly real GDP growth around NBER recession dates, we find evidence that they track each other. Additional statistical tests show that our uncertainty measure Granger causes real GDP growth in addition to other macroeconomic variables including industrial production and real consumption. Out-of-sample forecasting tests show that while our uncertainty measure adds predictive power to a simple forecast based on an AR(1) model for GDP growth, Diebold and Mariano (1995) tests indicate that there is no significant improvement in forecasts based on our uncertainty measure from those based solely on liquidity measures. We conclude that while in the cross-section of expected returns, investors are primarily concerned with this uncertainty risk, when forecasting economics variables there is no statistical difference between the accuracy of forecasts based on our uncertainty measure and those based on liquidity measures, although it does outperform the Roll (1984) liquidity measure.

## 3.5 Tables and Figures

Table 3.1: Contemporaneous Correlations.
Quarterly measures of liquidity (LIQ), volatility (VOL), and what we term "uncertainty" (UNC) are derived from liquidity and volatility measures calculated from daily CRSP data. The sample includes 5281 NYSE stocks from January 1947 to December 2012. The macroeconomic series span 1947Q1 to 2012Q4 and were obtained from the FRED as provided by the Federal Reserve Bank of St. Louis. p-values are listed in parentheses beneath the correlation coefficients.

| | UNC | LIQ | VOL | Amihud | Roll | Term | dCred | dGDP | dUE | dCons | dInv |
|---|---|---|---|---|---|---|---|---|---|---|---|
| LIQ | -0.80 | | | | | | | | | | |
| | (0.00) | | | | | | | | | | |
| VOL | 0.53 | 0.04 | | | | | | | | | |
| | (0.00) | (0.51) | | | | | | | | | |
| Amihud | 0.15 | -0.14 | 0.04 | | | | | | | | |
| | (0.02) | (0.03) | (0.49) | | | | | | | | |
| Roll | 0.34 | -0.31 | 0.17 | 0.55 | | | | | | | |
| | (0.00) | (0.00) | (0.00) | (0.00) | | | | | | | |
| Term | -0.15 | 0.14 | 0.03 | -0.13 | 0.03 | | | | | | |
| | (0.02) | (0.02) | (0.61) | (0.05) | (0.66) | | | | | | |
| dCred | 0.34 | -0.52 | -0.15 | -0.02 | 0.18 | -0.12 | | | | | |
| | (0.00) | (0.00) | 0.10) | (0.83) | (0.18) | (0.17) | | | | | |
| dGDP | -0.18 | 0.20 | -0.03 | -0.23 | -0.32 | 0.21 | -0.20 | | | | |
| | (0.00) | (0.00) | (0.58) | (0.00) | (0.00) | (0.00) | (0.02) | | | | |
| dUE | 0.14 | -0.10 | 0.09 | 0.28 | 0.31 | -0.15 | 0.20 | -0.58 | | | |
| | (0.03) | (0.12) | (0.15) | (0.00) | (0.00) | (0.02) | (0.03) | (0.00) | | | |
| dCons | -0.16 | 0.16 | -0.06 | -0.14 | -0.24 | 0.17 | -0.26 | 0.60 | -0.29 | | |
| | (0.01) | (0.01) | (0.32) | (0.02) | (0.00) | (0.01) | (0.00) | (0.00) | (0.00) | | |
| dInv | -0.18 | 0.18 | -0.01 | -0.19 | -0.29 | 0.26 | -0.18 | 0.78 | -0.49 | 0.21 | |
| | (0.00) | (0.00) | (0.82) | (0.00) | (0.00) | (0.00) | (0.05) | (0.00) | (0.00) | (0.00) | |
| dIP | -0.21 | 0.23 | -0.03 | -0.23 | -0.33 | 0.19 | -0.40 | 0.70 | -0.44 | 0.49 | 0.66 |
| | (0.00) | (0.00) | (0.67) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |

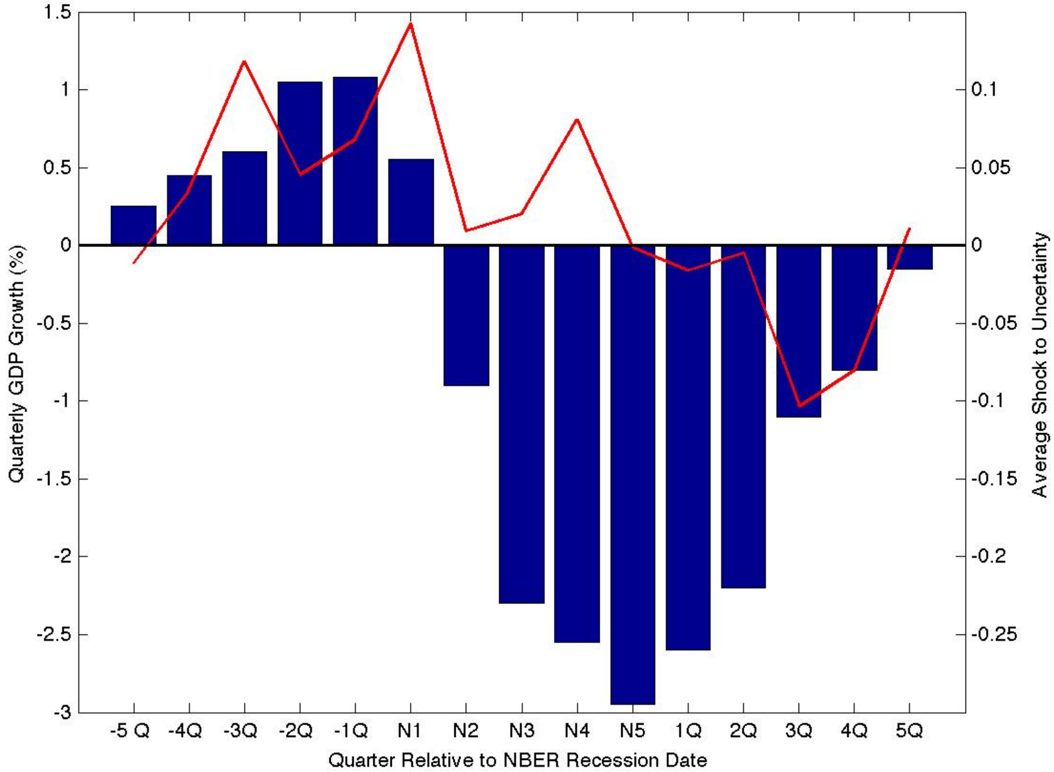


Figure 3.1: Average Quarterly real GDP growth.
We construct a window around each of the NBER recession start dates that spans 5 quarters prior to the recession, 5 quarters during the recession, and 5 quarters after the recession. We plot the accumulated average quarterly GDP growth in a bar chart. Also included is a plot of the average shock to our uncertainty measure (based on multiple liquidity and volatility measures) during the same quarters. We find that in the quarters leading up to a recession, there are positive shocks. Toward the end of the recession and during the start of the recovery, we see that on average there are negative shocks to uncertainty.

Table 3.2: In-Sample Predictive Regressions.
Quarterly measures of what we term "uncertainty" (UNC) are derived from liquidity and volatility measures calculated from daily CRSP data. The sample includes 5281 NYSE stocks from January 1947 to December 2012. The macroeconomic series span 1947Q1 to 2012Q4 and were obtained from the FRED as provided by the Federal Reserve Bank of St. Louis. The estimated models are of the form $y_{t+1} = \alpha + \beta UNC_t + \gamma' X_t + u_{t+1}$ where $UNC_t$ is our uncertainty measure and $X_t$ contains additional regressors including the lagged dependent variable. The Newey and West (1987) adjusted t-stats with 4 lags are presented in parentheses beneath the parameter estimates.

| $y_{t+1}$ | $\hat{\alpha}$ | $\hat{\beta}$ | $\hat{\gamma}^y$ | $\hat{\gamma}^{Term}$ | $R^2$ |
|---|---|---|---|---|---|
| dGDP | 0.006 | -0.010 | 0.212 | | 0.08 |
| | (7.33) | (-2.38) | (3.40) | | |
| | 0.005 | -0.008 | 0.203 | 0.001 | 0.10 |
| | (3.88) | (-1.92) | (2.82) | (2.18) | |
| dIP | 0.007 | -0.023 | 0.142 | | 0.06 |
| | (4.24) | (-2.81) | (2.35) | | |
| | 0.003 | -0.019 | 0.113 | 0.003 | 0.09 |
| | (1.27) | (-2.59) | (1.47) | (2.32) | |
| dUE | 0.001 | 0.063 | 0.234 | | 0.07 |
| | (0.23) | (2.97) | (3.32) | | |
| | 0.021 | 0.042 | 0.253 | -0.013 | 0.13 |
| | (2.70) | (2.14) | (3.25) | (-3.90) | |
| dCons | 0.006 | -0.006 | 0.320 | | 0.13 |
| | (7.04) | (-2.16) | (4.58) | | |
| | 0.005 | -0.006 | 0.263 | 0.001 | 0.13 |
| | (5.30) | (-2.03) | (3.12) | (1.72) | |
| dInv | 0.009 | -0.054 | 0.110 | | 0.05 |
| | (2.57) | (-2.75) | (1.78) | | |
| | -0.001 | -0.043 | 0.071 | 0.007 | 0.09 |
| | (-0.26) | (-2.13) | (1.06) | (2.90) | |

Table 3.3: Granger Causality.
Quarterly measures of what we term "uncertainty" (UNC) are derived from liquidity and volatility measures calculated from daily CRSP data. The sample includes 5281 NYSE stocks from January 1947 to December 2012. The macroeconomic series span 1947Q1 to 2012Q4 and were obtained from the FRED as provided by the Federal Reserve Bank of St. Louis. A VAR specification is used in the tests of Granger causality where the number of included lags was chosen using the Bayesian Information Criterion (BIC). Here the null is that there is no Granger causality between the variables so a statistically significant test rejects the null of no Granger causality.

| Test | Entire Sample 1947-2012 | First Half 1947-1979 | Second Half 1980-2012 |
|---|---|---|---|
| $H_0 : dGDP \nrightarrow UNC$ | | | |
| $\chi^2$ | 0.06 | 0.26 | 0.40 |
| $p$-value | 0.81 | 0.61 | 0.53 |
| $H_0 : UNC \nrightarrow dGDP$ | | | |
| $\chi^2$ | 6.79** | 8.34** | 2.71* |
| $p$-value | 0.01 | 0.00 | 0.10 |
| $H_0 : dIP \nrightarrow UNC$ | | | |
| $\chi^2$ | 2.62 | 0.03 | 4.65** |
| $p$-value | 0.11 | 0.86 | 0.03 |
| $H_0 : UNC \nrightarrow dIP$ | | | |
| $\chi^2$ | 9.47** | 6.11** | 5.71** |
| $p$-value | 0.00 | 0.01 | 0.02 |
| $H_0 : dCons \nrightarrow UNC$ | | | |
| $\chi^2$ | 1.54 | 1.76 | 0.39 |
| $p$-value | 0.21 | 0.19 | 0.53 |
| $H_0 : UNC \nrightarrow dCons$ | | | |
| $\chi^2$ | 5.54** | 6.84** | 1.66 |
| $p$-value | 0.02 | 0.01 | 0.20 |
| $H_0 : dUE \nrightarrow UNC$ | | | |
| $\chi^2$ | 0.35 | 1.72 | 0.23 |
| $p$-value | 0.55 | 0.19 | 0.63 |
| $H_0 : UNC \nrightarrow dUE$ | | | |
| $\chi^2$ | 4.97** | 5.99** | 0.78 |
| $p$-value | 0.03 | 0.01 | 0.38 |

Table 3.4: Out-of-Sample Real GDP Growth Forecast Performance.
Quarterly measures of what we term "uncertainty" (UNC) are derived from liquidity and volatility measures calculated from daily CRSP data. The sample includes 5281 NYSE stocks from January 1947 to December 2012. The macroeconomic series span 1947Q1 to 2012Q4 and were obtained from the FRED as provided by the Federal Reserve Bank of St. Louis. This table includes tests between nested and non-nested models. The Diebold-Mariano (1995) test statistic was used to compare non-nested models while the ENC-NEW of Clark and McCracken (2001) was used for the nested model comparisons. The non-nested models always include the lagged GDP growth as well as one of the measures of either uncertainty or liquidity. The null hypothesis for the DM test is that of equal MSFE with a one-sided alternative that Model 2 has a lower MSFE than Model 1. 5% and 10% significance are denoted with a ** and * respectively.

Non-Nested Tests (forecasting GDP growth)

| | Model 2 | | |
|---|---|---|---|
| Model 1 | UNC | LIQ | Amihud |
| LIQ | -0.38 | | |
| Amihud | 0.26 | 0.49 | |
| Roll | 1.53* | 1.75* | 1.55* |

Nested Tests (AR(1))

| Unrestricted Model | Restricted Model | ENC-NEW | Unrestricted Model | Restricted Model | ENC-NEW |
|---|---|---|---|---|---|
| UNC, dGDP | dGDP | 32.49** | UNC, LIQ, dGDP | LIQ, dGDP | 1.26 |
| LIQ, dGDP | dGDP | 37.11** | UNC, dCons | dCons | -0.83 |
| UNC, dIP | dIP | 56.28** | UNC, dUE | dUE | 97.25** |

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# Biography

My name is Benjamin Arthur Carlston. I was born on March 12, 1983 in Walnut Creek, California. I received my undergraduate degree in Economics with a second major in Mathematics at Brigham Young University. All of my graduate studies have been completed in Economics at Duke University.